Example-Based Subspace Stress Analysis for Interactive Shape Design

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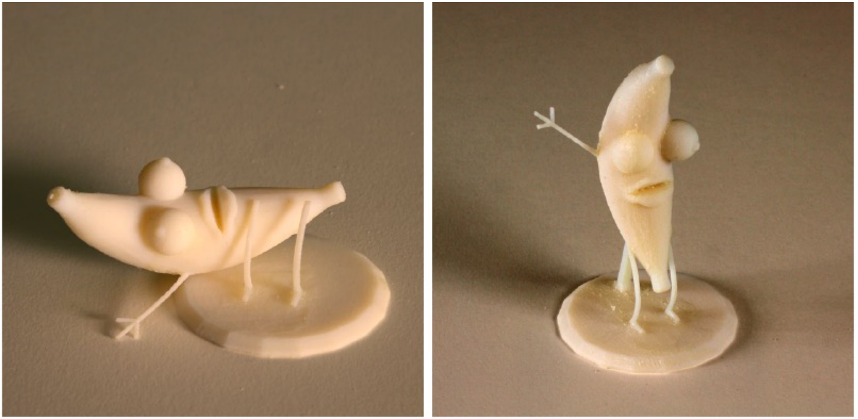
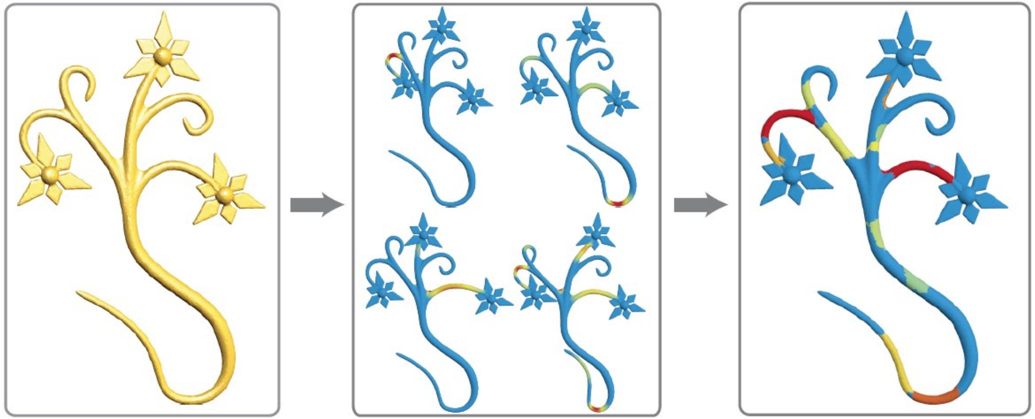
Stress Analysis

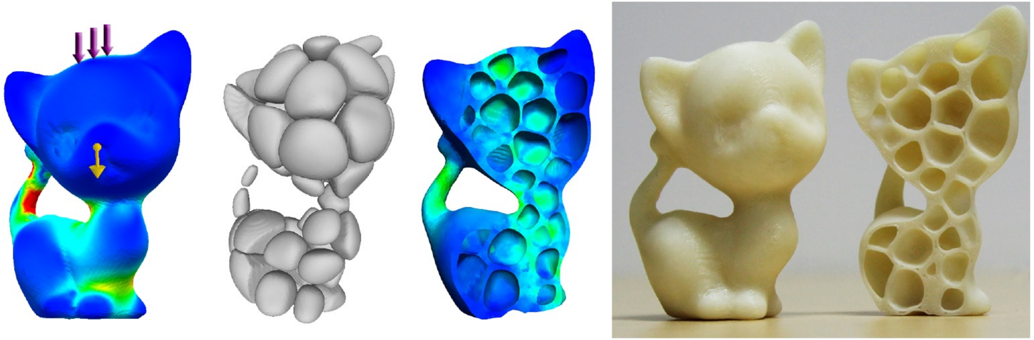


[Stava et al. 2012]



[Zhou et al. 2013]

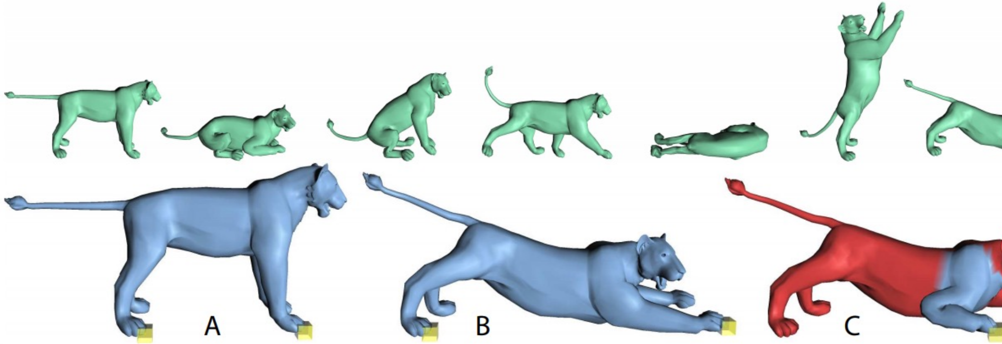
[Wang et al. 2013]



[Lu et al. 2014]

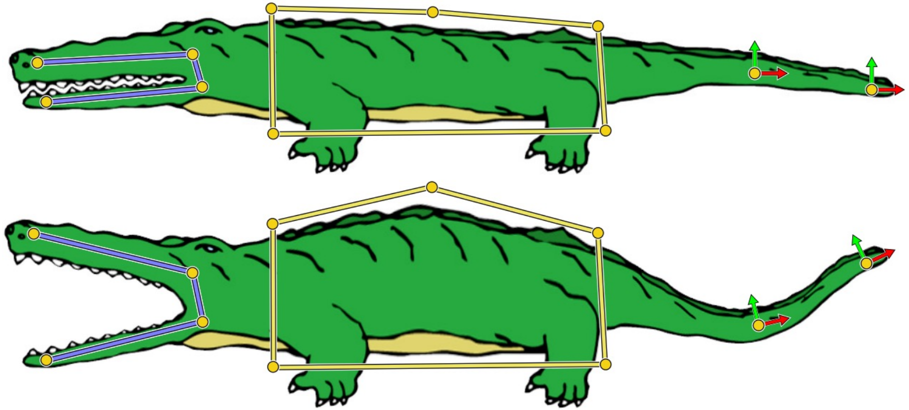
Subspace Shape Deformation

[Lewis et al. 2000]



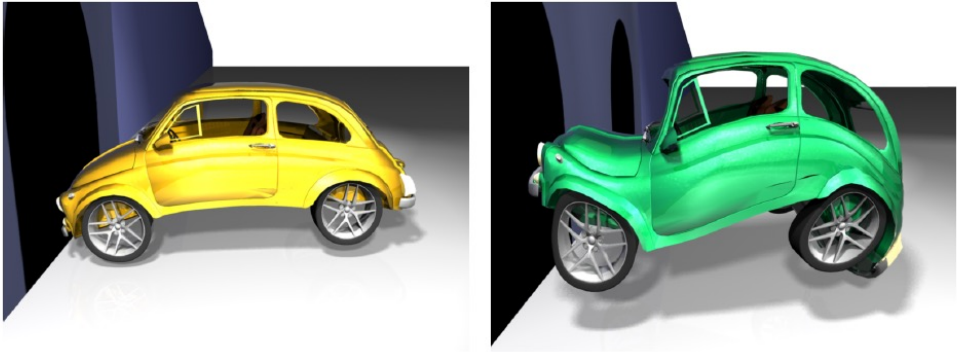
[Sumner et al. 2005]

[Weber et al. 2007]

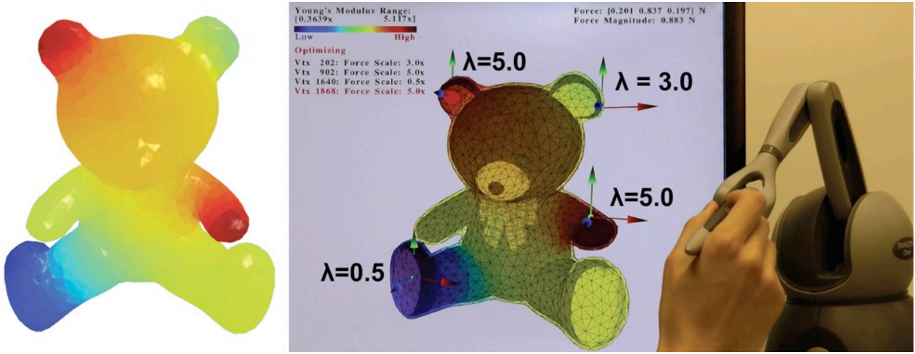


[Jacobson et al. 2011]

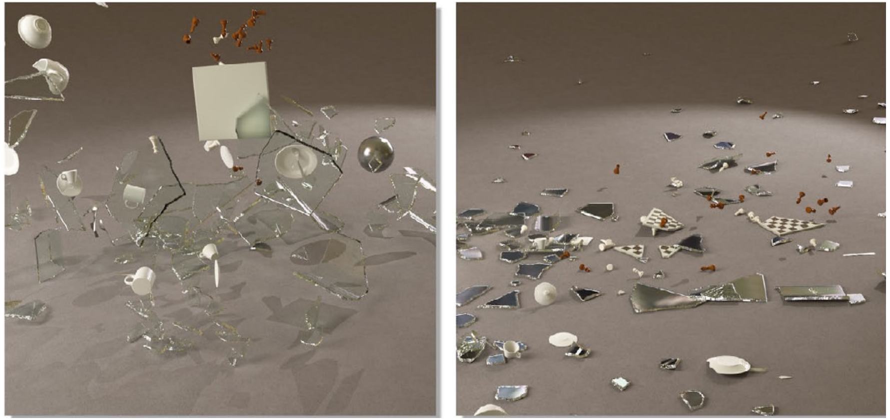
Subspace Dynamic Simulation



[Martin et al. 2011]



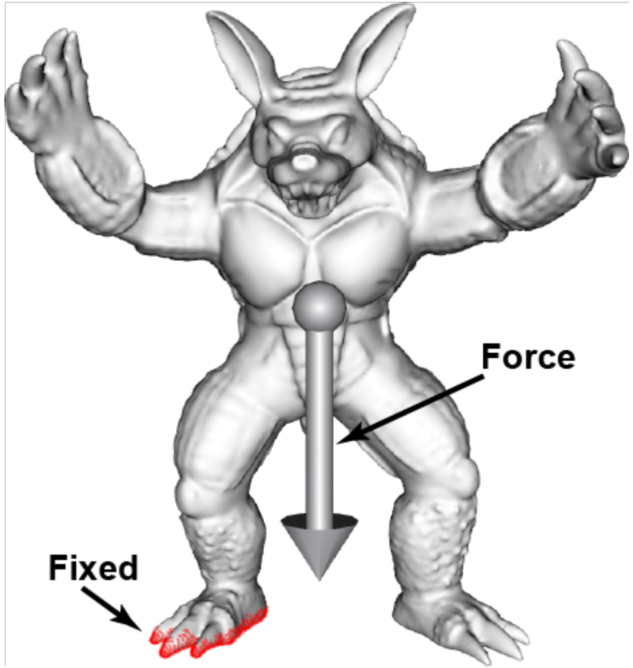
[Xu et al. 2015]

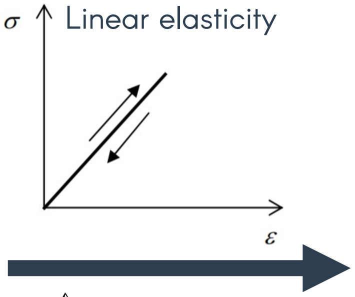


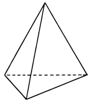
[Zheng et al. 2010]

[Chai et al. 2014]

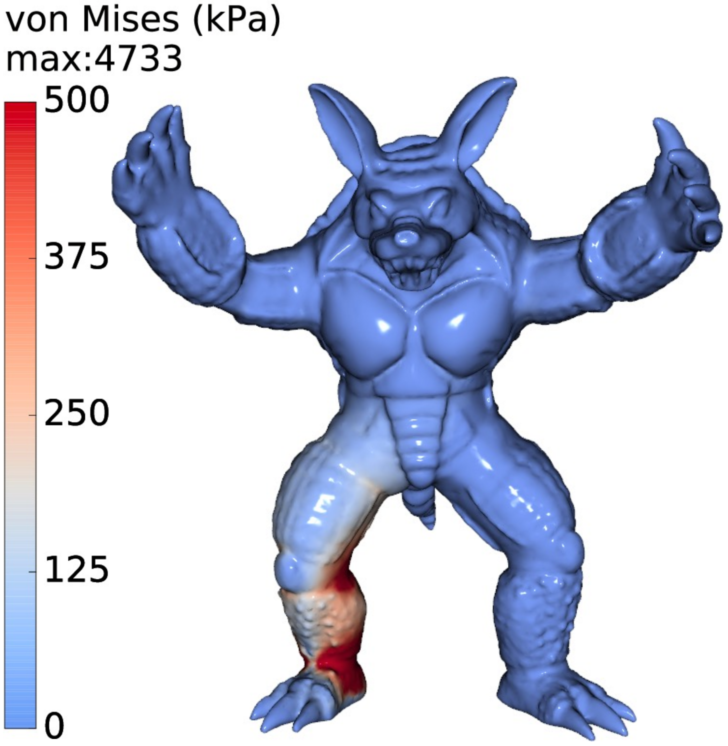
Background Concepts





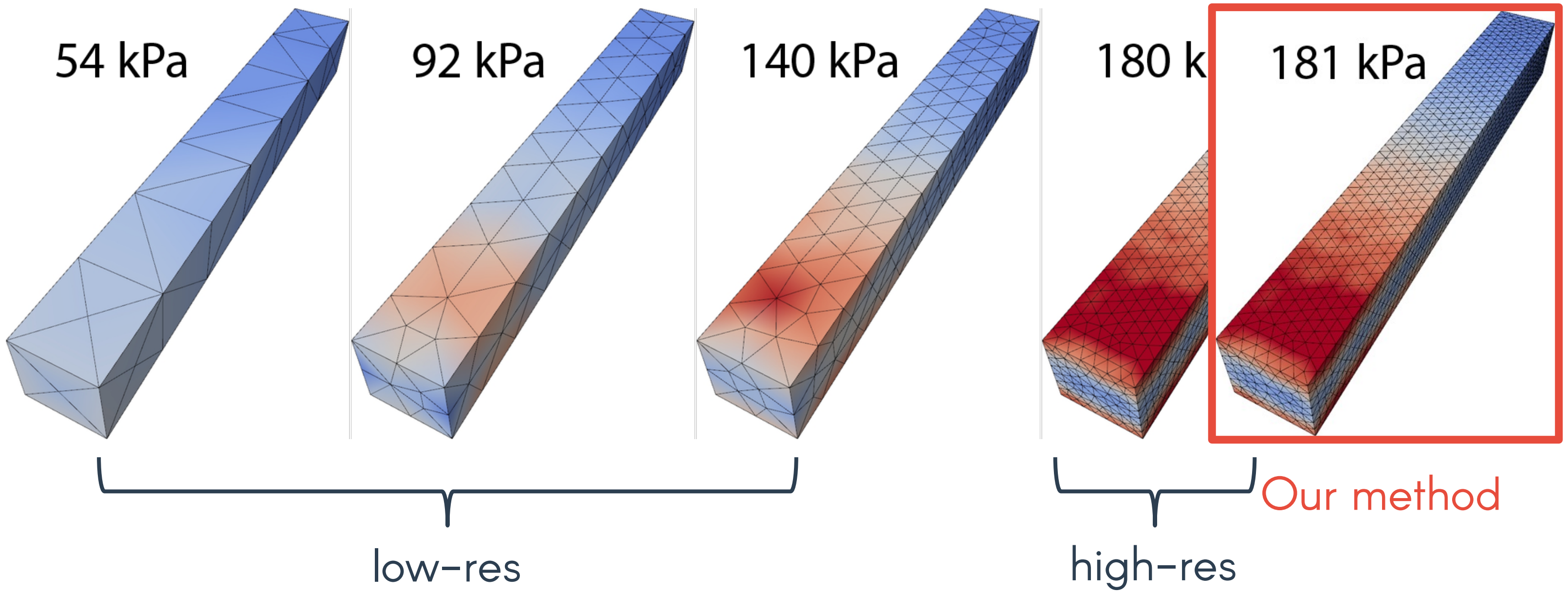


8 minutes



Reduced Stress Analysis

Low resolution mesh



Reduced Stress Analysis

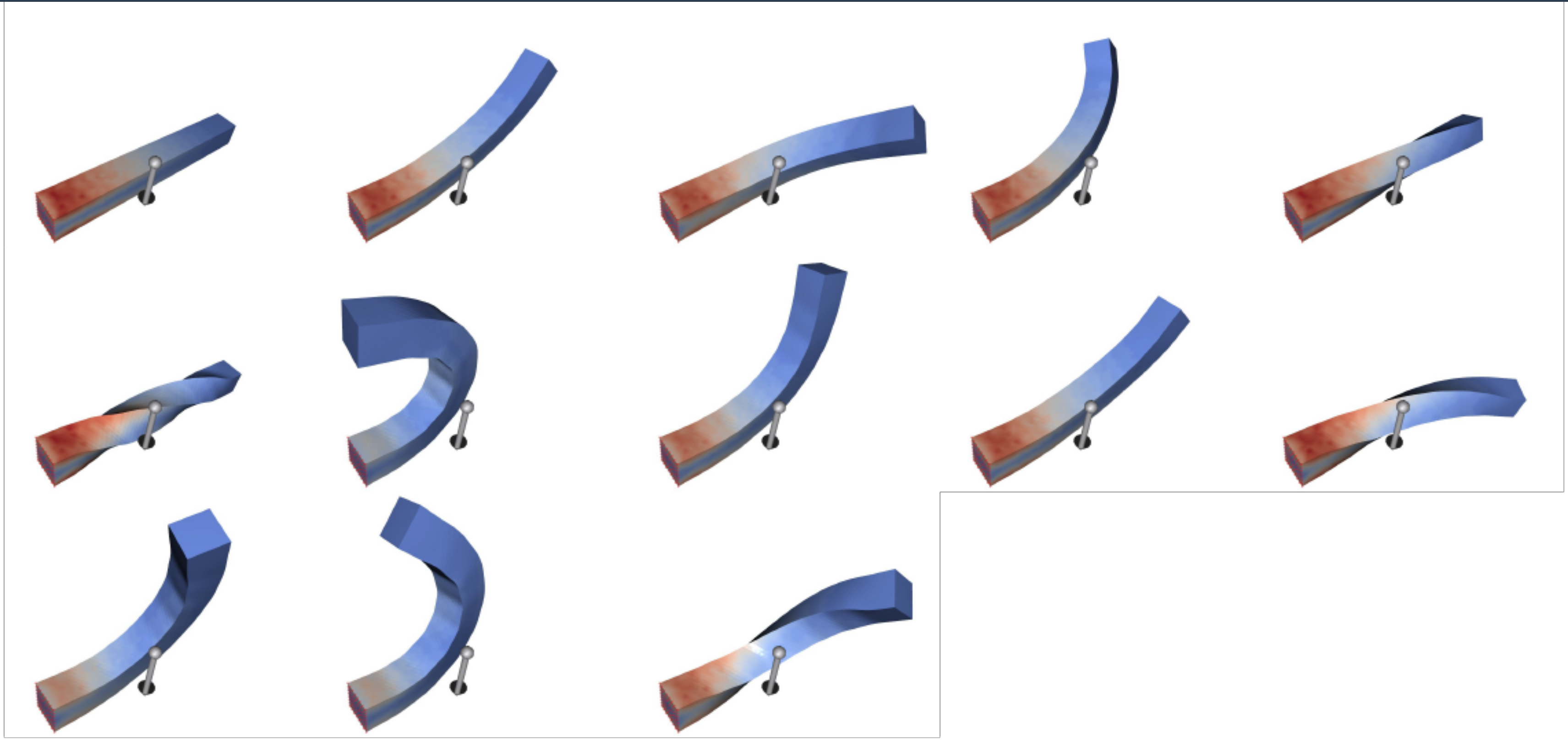
Rest shapes correlated

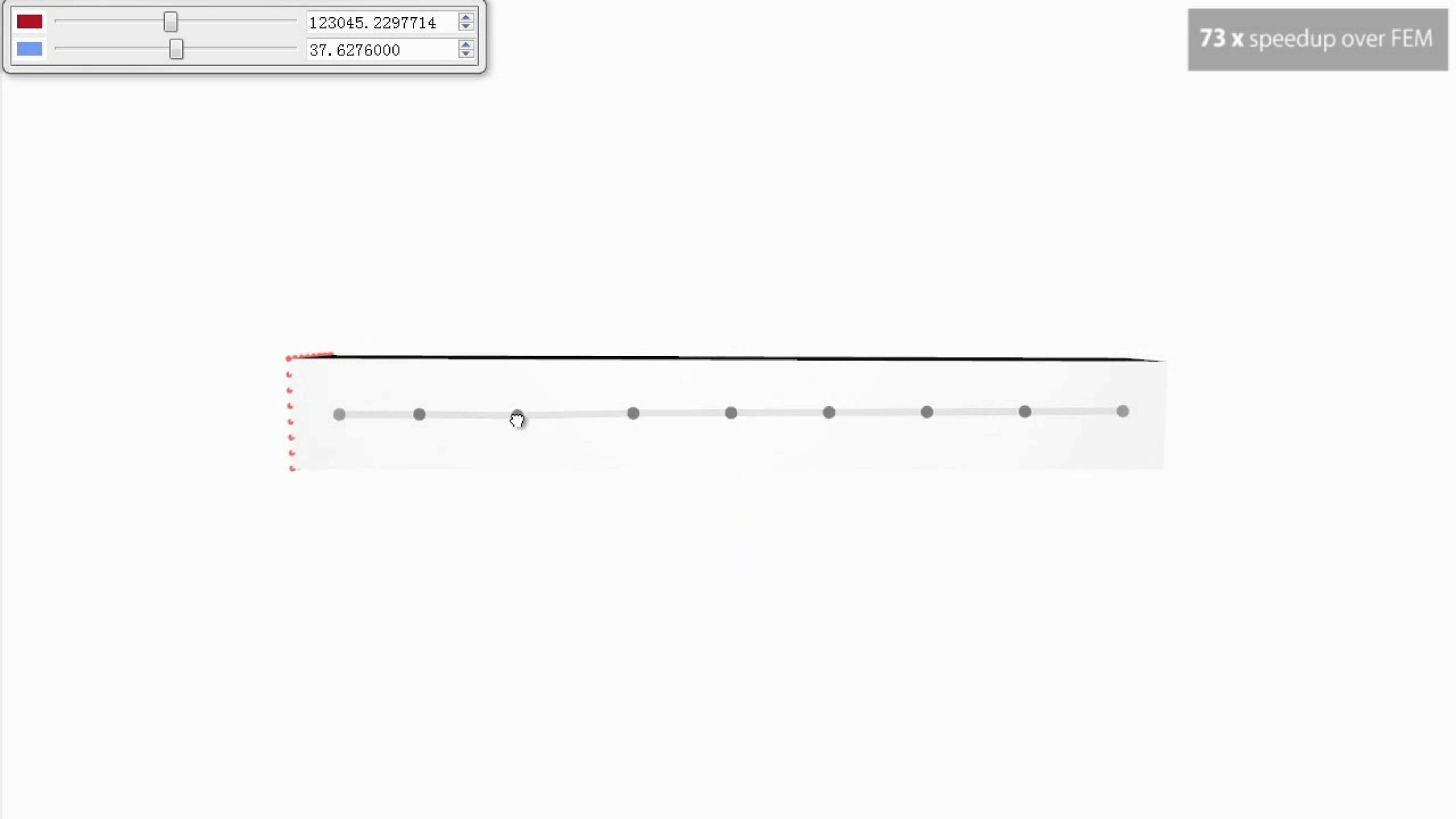
Two phase strategy

Subspace Precomputation

Runtime Estimation

Reduced Stress Analysis




123045.2297714
37.6276000
73 x speedup over FEM

Reduced Stress Analysis

Subspace
Precomputation

Runtime
Estimation

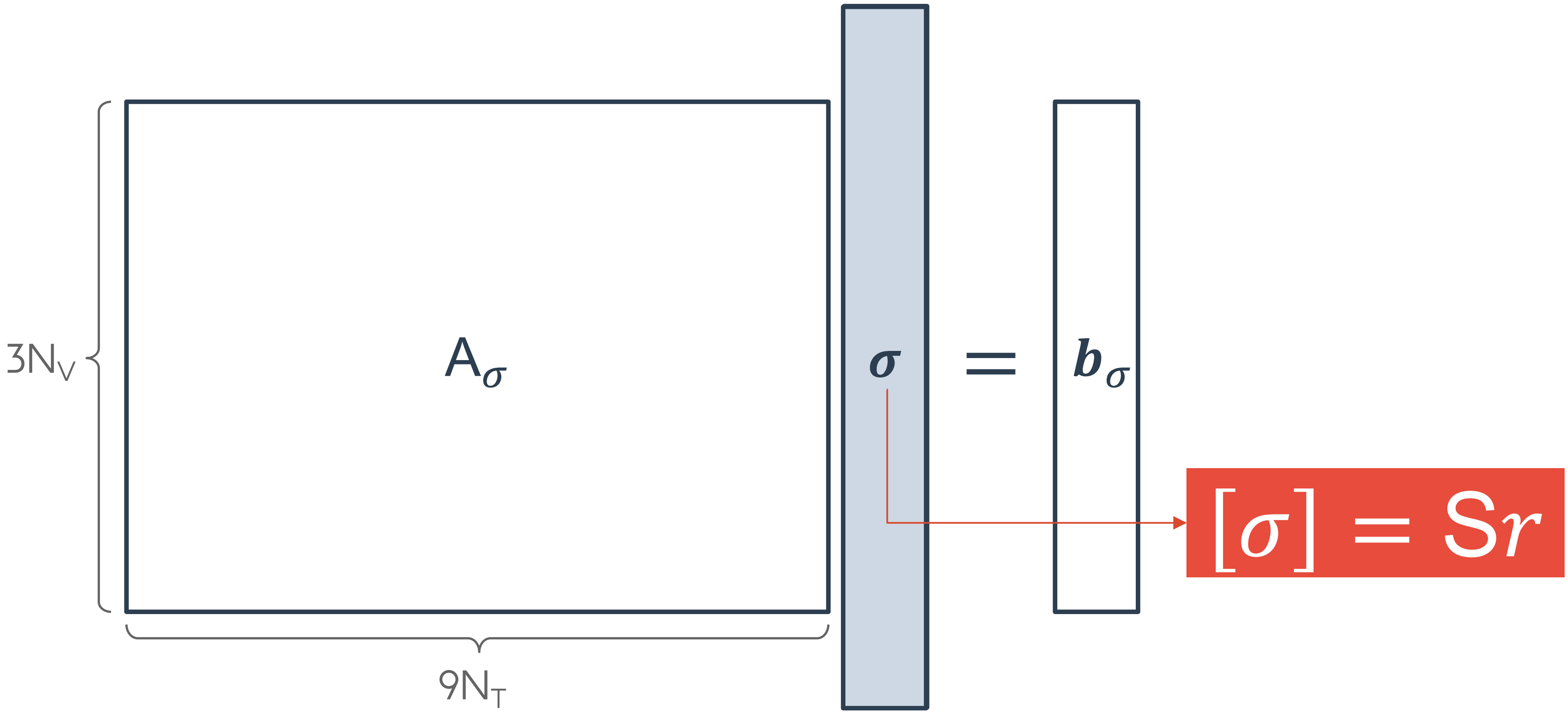
Reduced Stress Analysis

Standard FEM discretizes the displacement, $\boldsymbol{u}$

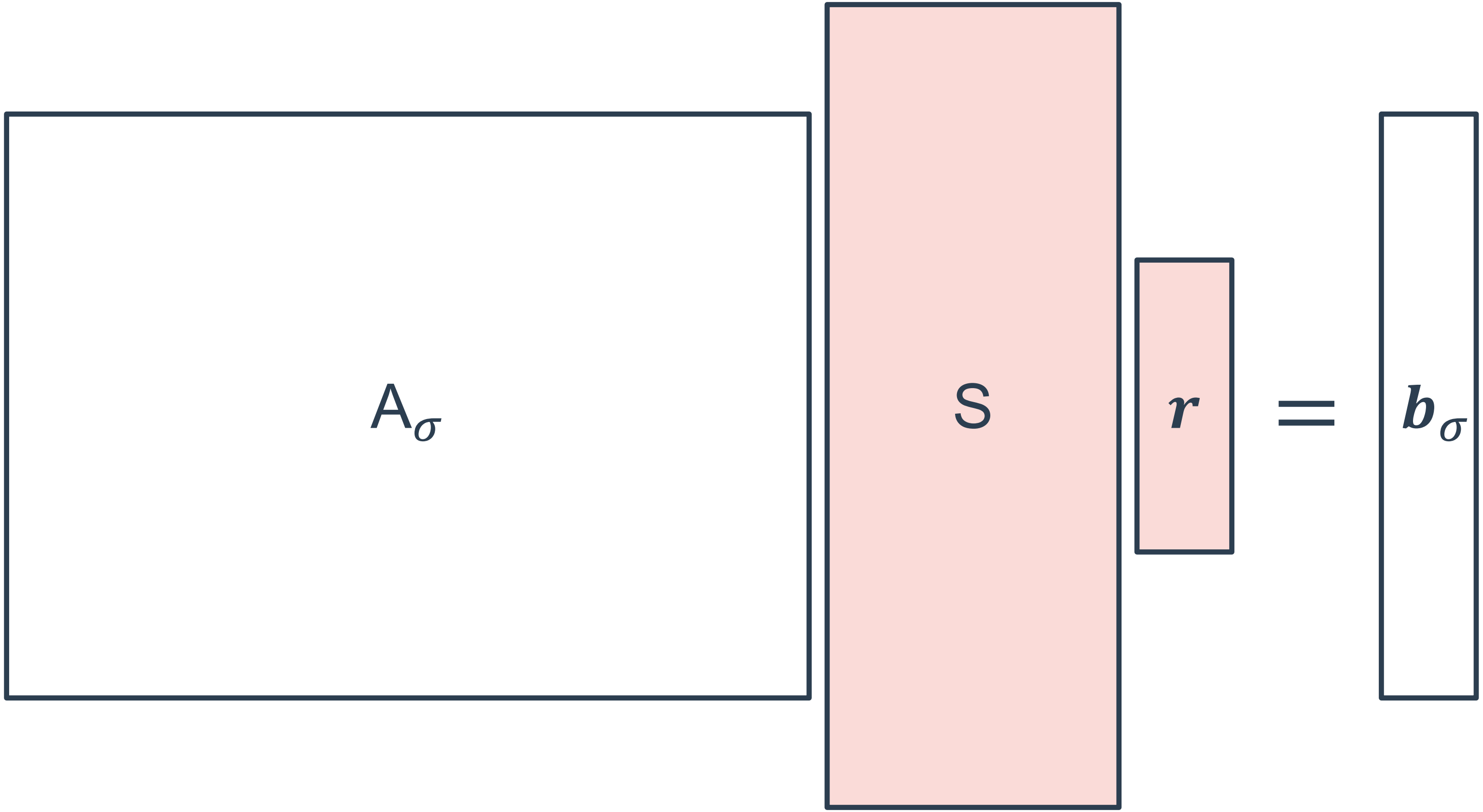
Instead, discretizing the weak form w.r.t. stress, $\boldsymbol{\sigma}$

$$\mathrm{A}_{\sigma}[\boldsymbol{\sigma}] = \boldsymbol{b}_{\sigma}$$

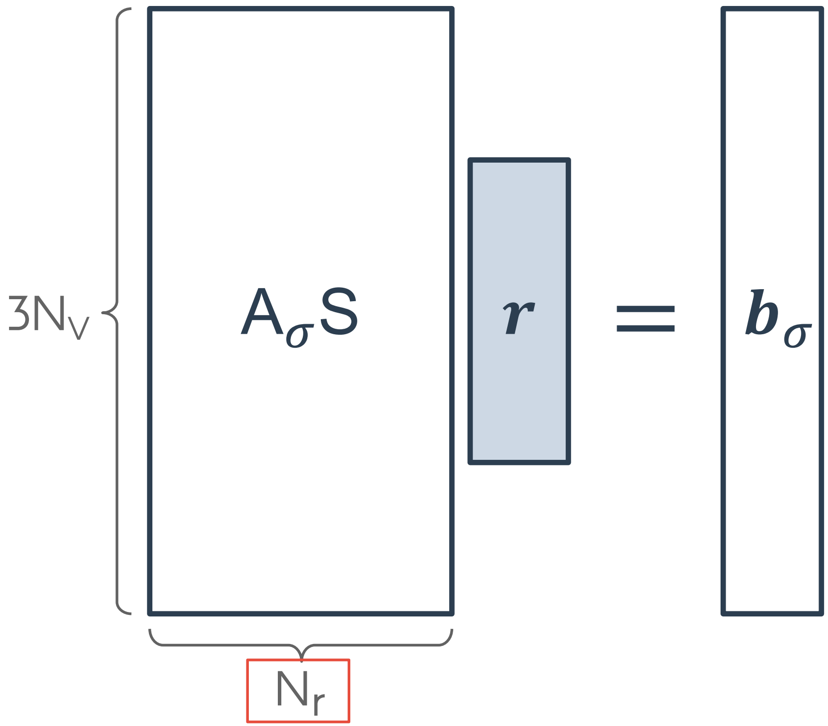
Reduced Stress Analysis



Reduced Stress Analysis



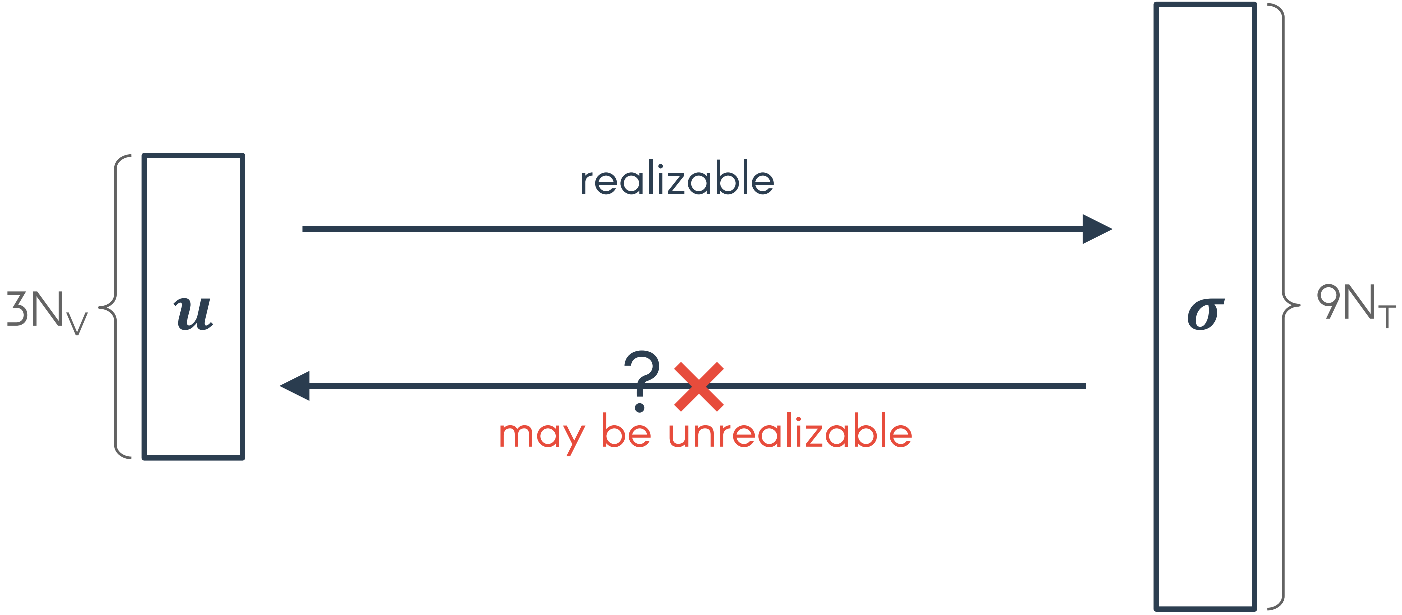
Reduced Stress Analysis



Small-scale least-squares problem

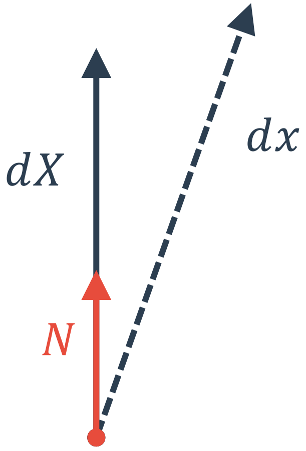
Reduced Stress Analysis

Physically realizable stress?



Reduced Stress Analysis

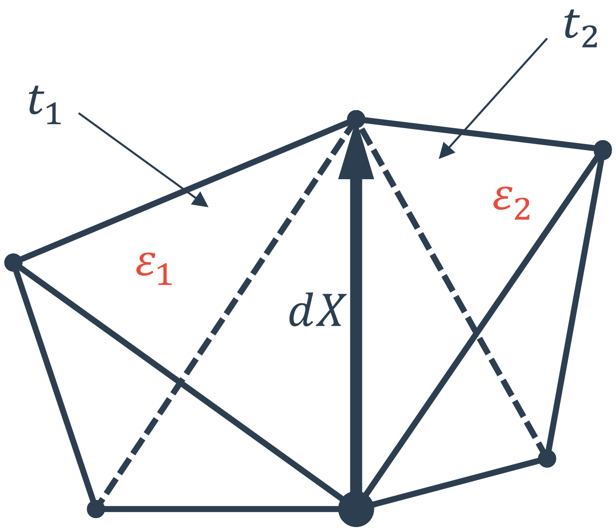
Regularization



$$N^T \boldsymbol{\varepsilon} N = \frac{\|dx\|_2 - \|dX\|_2}{\|dX\|_2}$$

Reduced Stress Analysis

Regularization



$$N^T \varepsilon_1 N = N^T \varepsilon_2 N$$

Reduced Stress Analysis

Quadratic minimization

$$\min_{\boldsymbol{r}} ||(\mathrm{A}_\sigma \mathrm{S})\boldsymbol{r} - \boldsymbol{b}_\sigma||_2^2 + \alpha E_r(\boldsymbol{r})$$

Reduced Stress Analysis

Quadratic minimization

Weight factor

$$\min_{\boldsymbol{r}} \underbrace{\|(\mathrm{A}_\sigma \mathrm{S})\boldsymbol{r} - \boldsymbol{b}_\sigma\|_2^2}_{\text{Force term}} + \alpha \underbrace{E_r(\boldsymbol{r})}_{\text{Regularization term}}$$

Force term

Regularization term

Reduced Stress Analysis

Subspace Precomputation

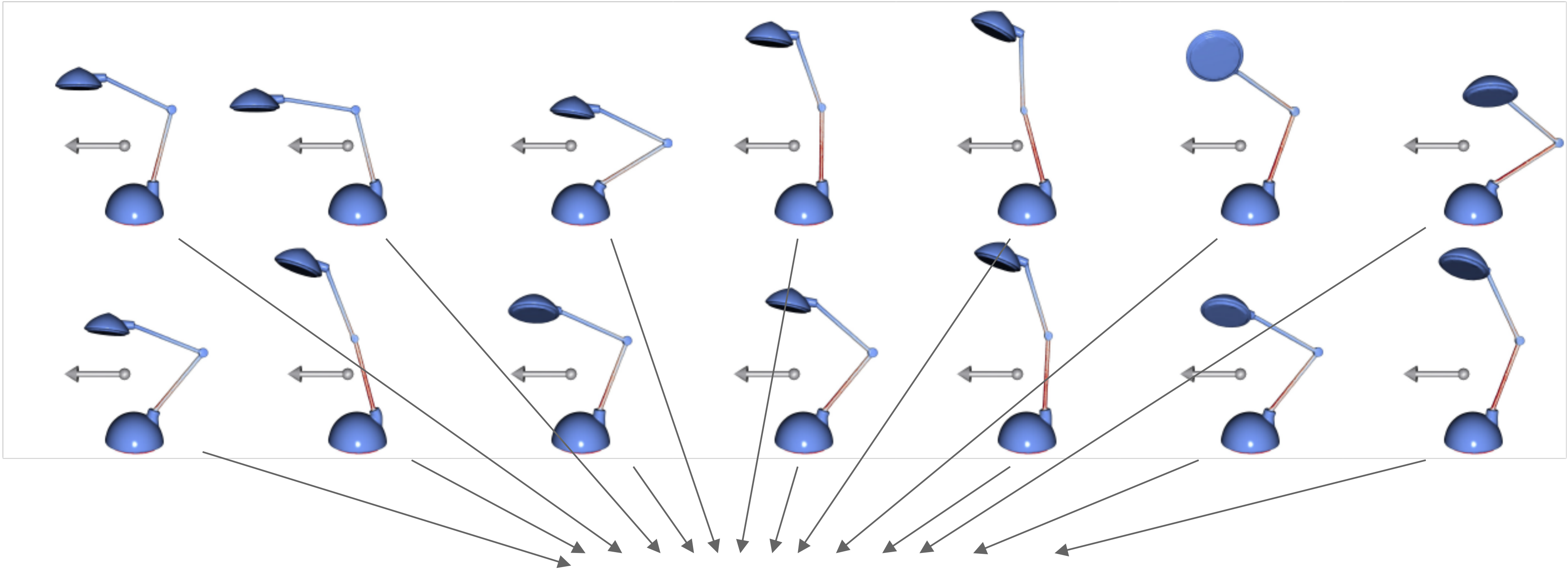
Runtime Estimation

Construction of Stress Basis

How to compute?

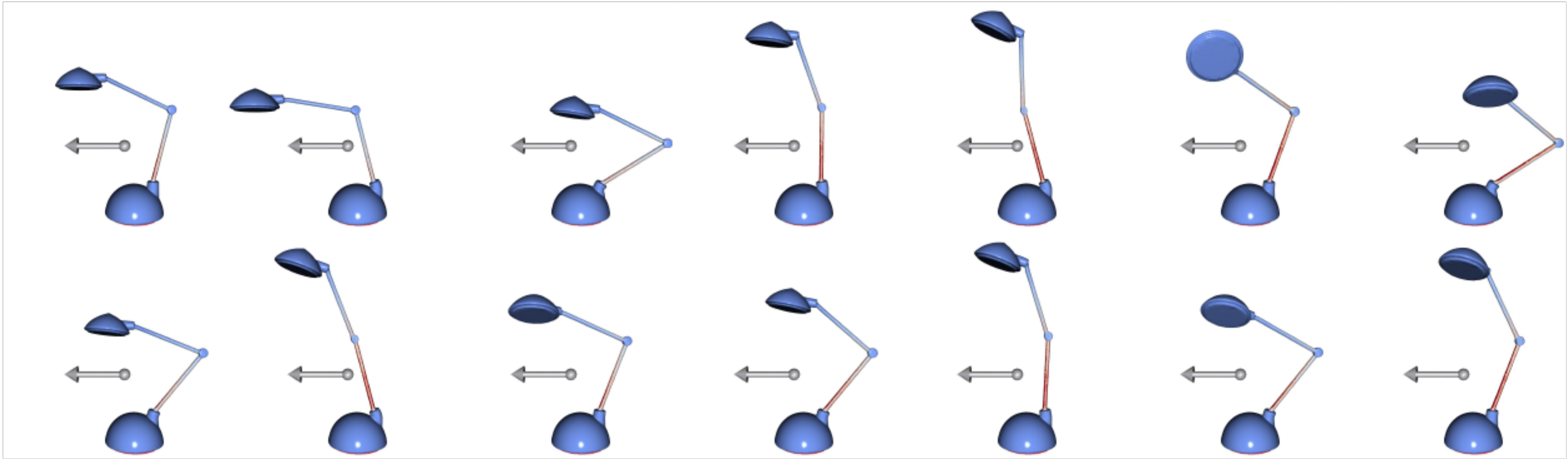
$$[\sigma] = \mathsf{S}r$$

Construction of Stress Basis



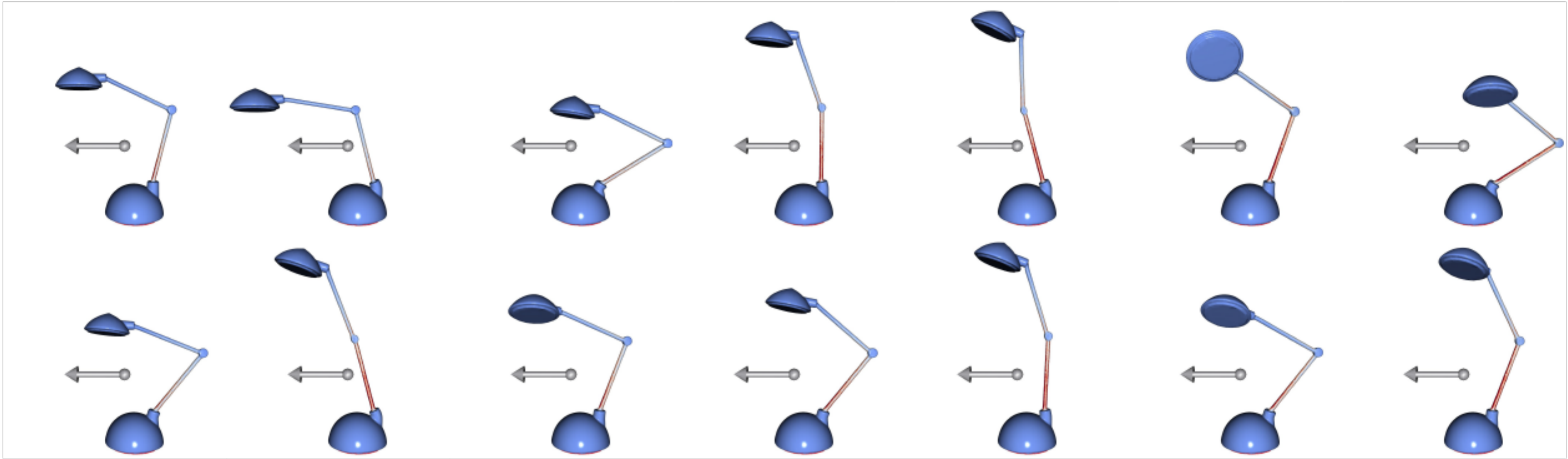
$$B = [\sigma_1, \sigma_2, \ldots, \sigma_s]$$

Construction of Stress Basis



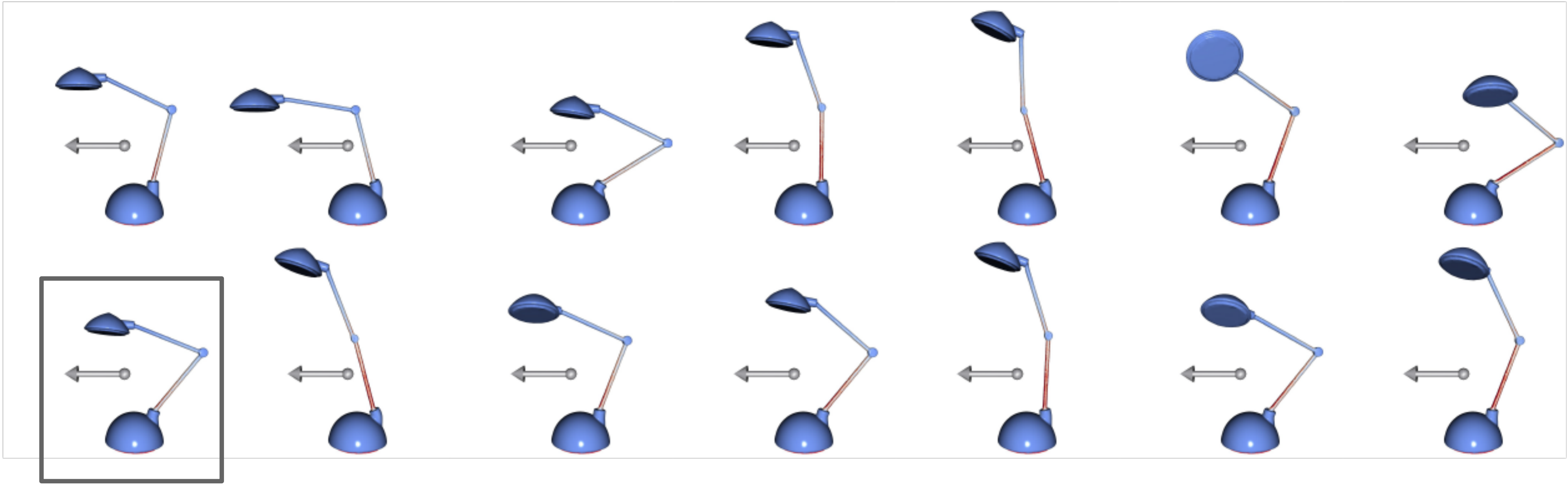
$$S = \mathrm{PCA}(B)$$

Construction of Stress Basis



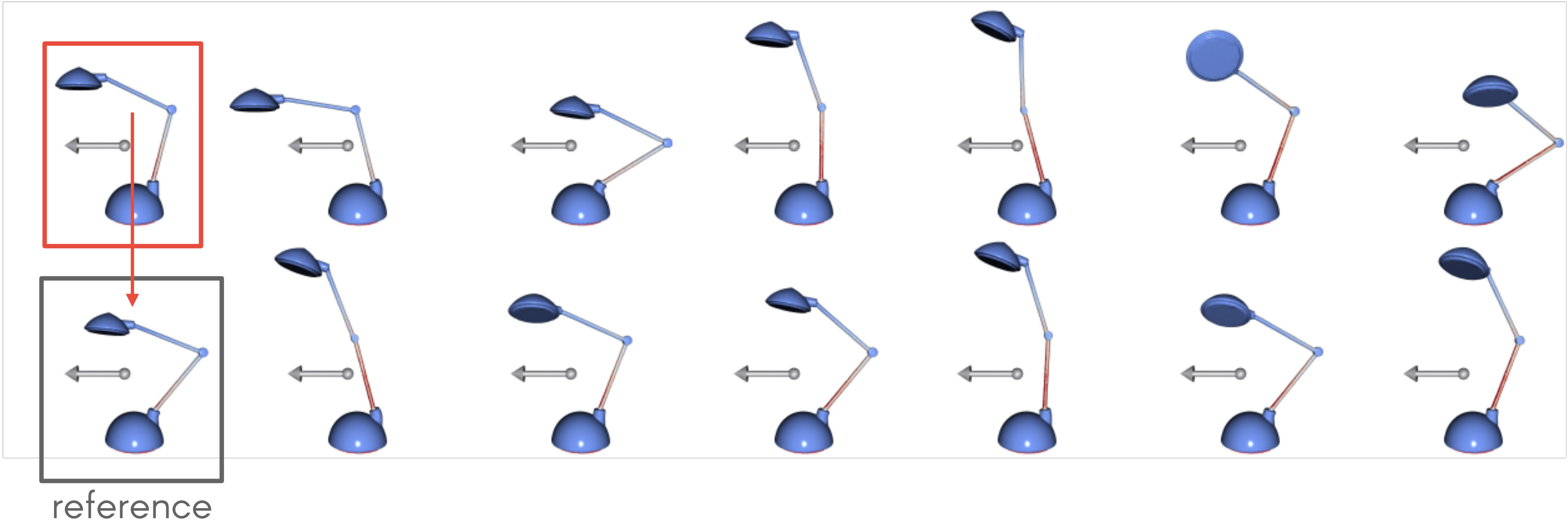
$$S = \mathrm{PCA}(B) \rightarrow pb(B)$$

Construction of Stress Basis



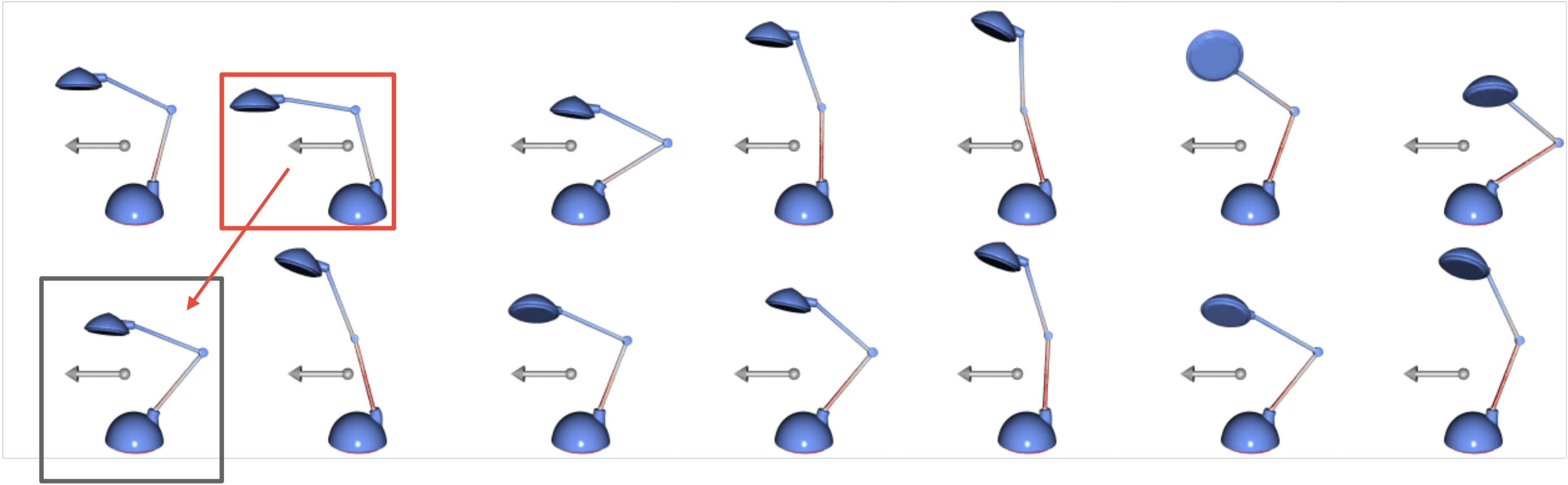
$$S = \mathrm{PCA}([\sigma_1, \sigma_2, \ldots, \sigma_s])$$

Construction of Stress Basis



$$S = \mathrm{PCA}([pb(\sigma_1), \sigma_2, \ldots, \sigma_s])$$

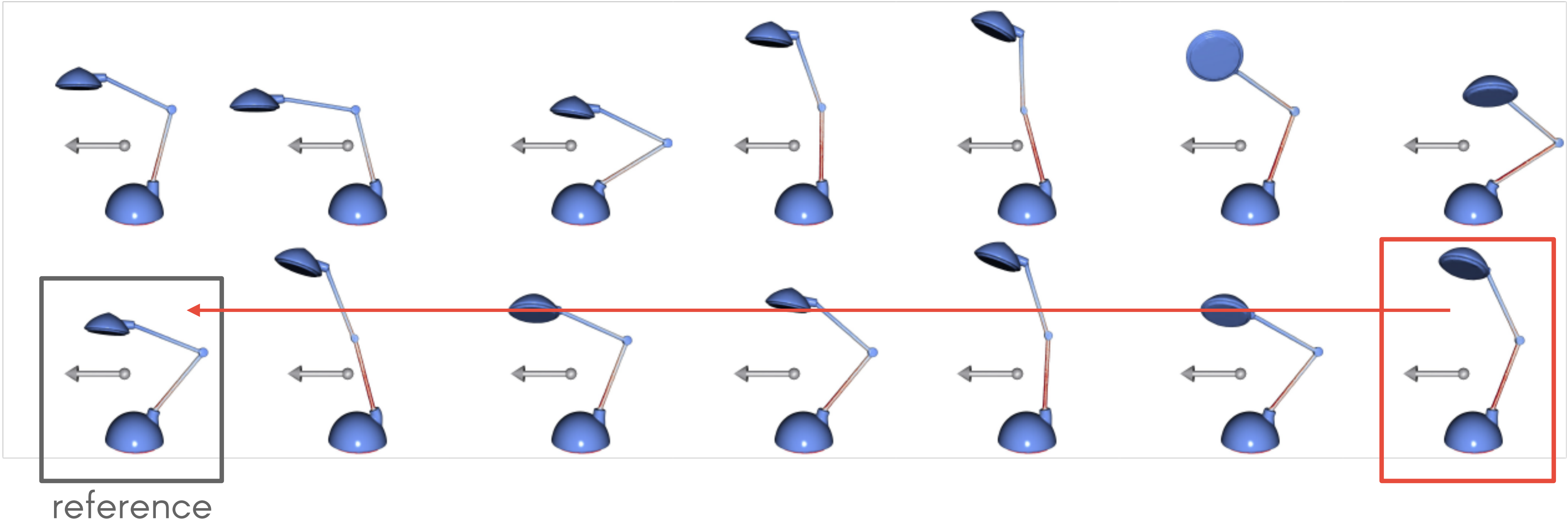
Construction of Stress Basis



reference

$$S = \mathrm{PCA}([pb(\sigma_1), pb(\sigma_2), \ldots, \sigma_s])$$

Construction of Stress Basis



$$S = \mathrm{PCA}([pb(\sigma_1), pb(\sigma_2), \ldots, pb(\sigma_s)])$$

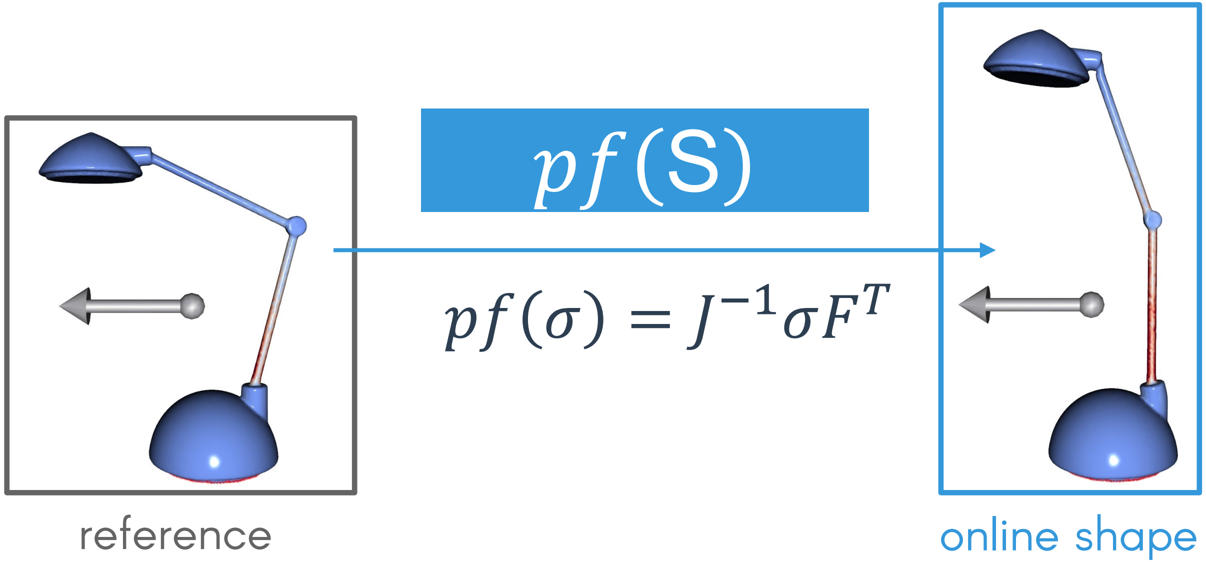
Construction of Stress Basis

Stress tensor pull-back (inspired from the first PK tensor)

$$pb(\sigma) = J\sigma F^{-T}$$

Construction of Stress Basis

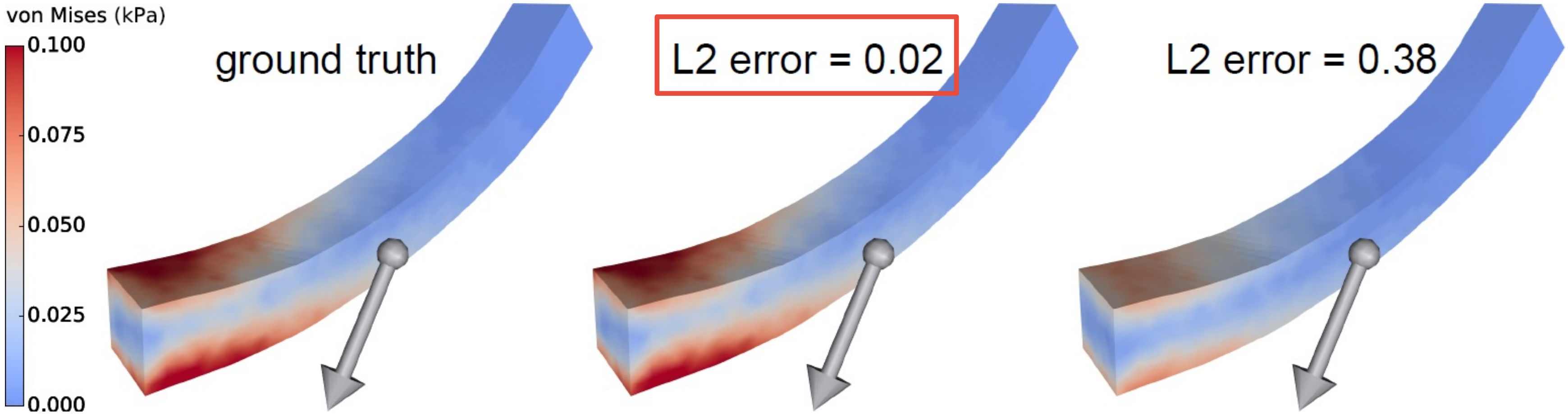
Stress tensor push-forward



$$pf(\sigma) = J^{-1}\sigma F^{T}$$

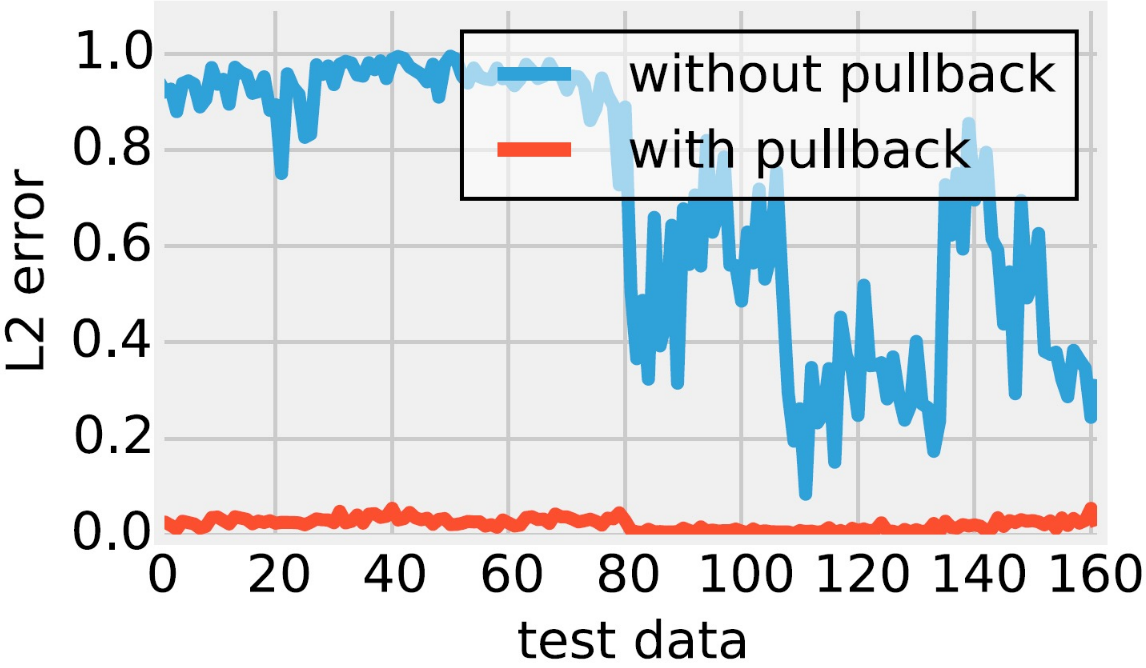
Construction of Stress Basis

Efficacy of pull-back and push-forward



Construction of Stress Basis

Efficacy of pull-back and push-forward



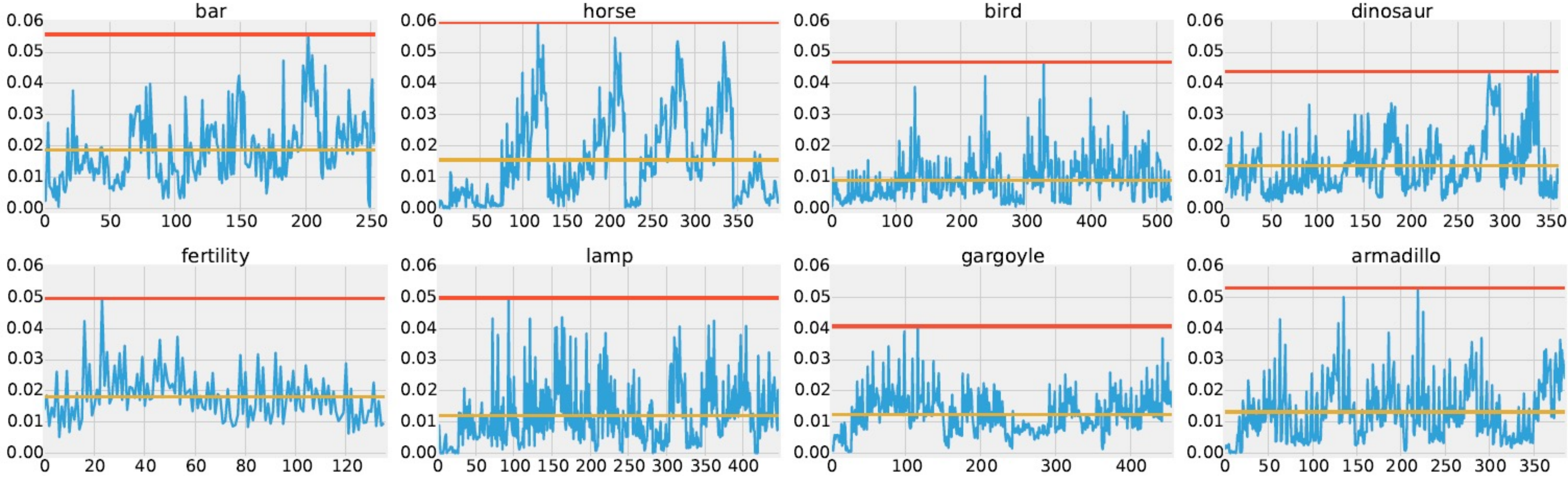
Results

Training and test data: 8 groups

Model	#Vertices	#Elements	#Order	#DoFs	#Train	#Test
bar	3,452	15,602	linear	10,356	39	254
horse	8,253	34,019	quadratic	167,034	21	399
bird	10,876	43,299	quadratic	216,072	27	525
dinosaur	14,029	57,830	linear	42,087	33	359
fertility	17,492	82,271	quadratic	377,667	18	136
lamp	20,428	94,184	linear	61,284	42	448
gargoyle	30,455	145,076	linear	91,365	33	457
armadillo	118,595	591,547	linear	355,785	33	387

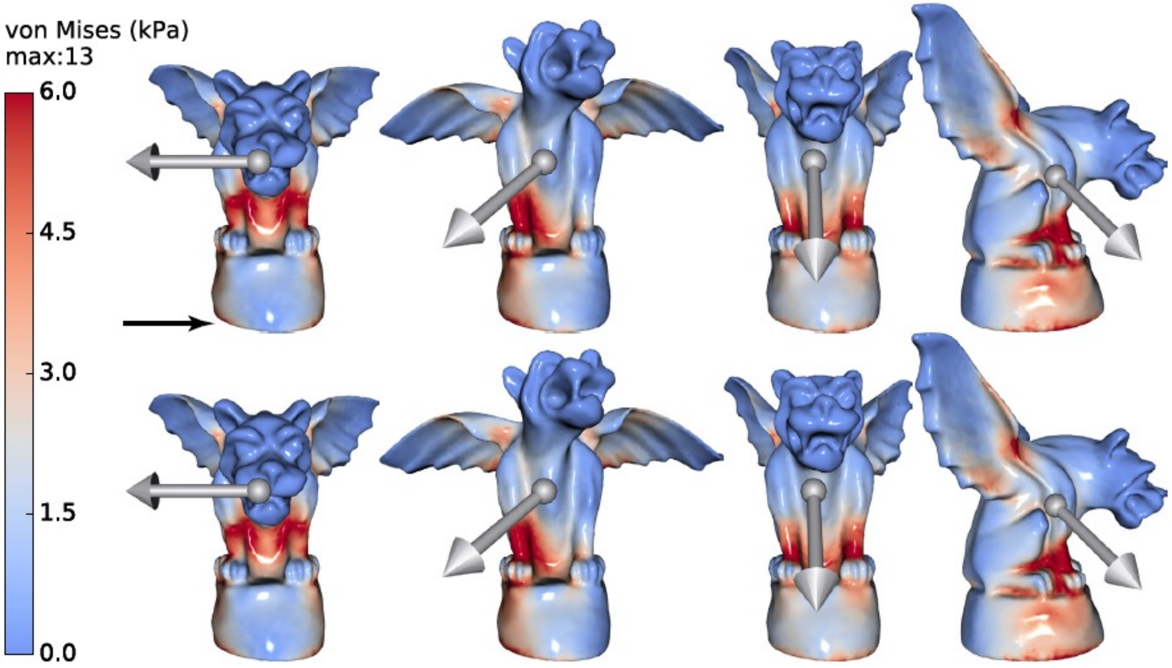
Results

Accuracy – L_2 errors: max < 6%, average < 2%



Results

Accuracy – visualization

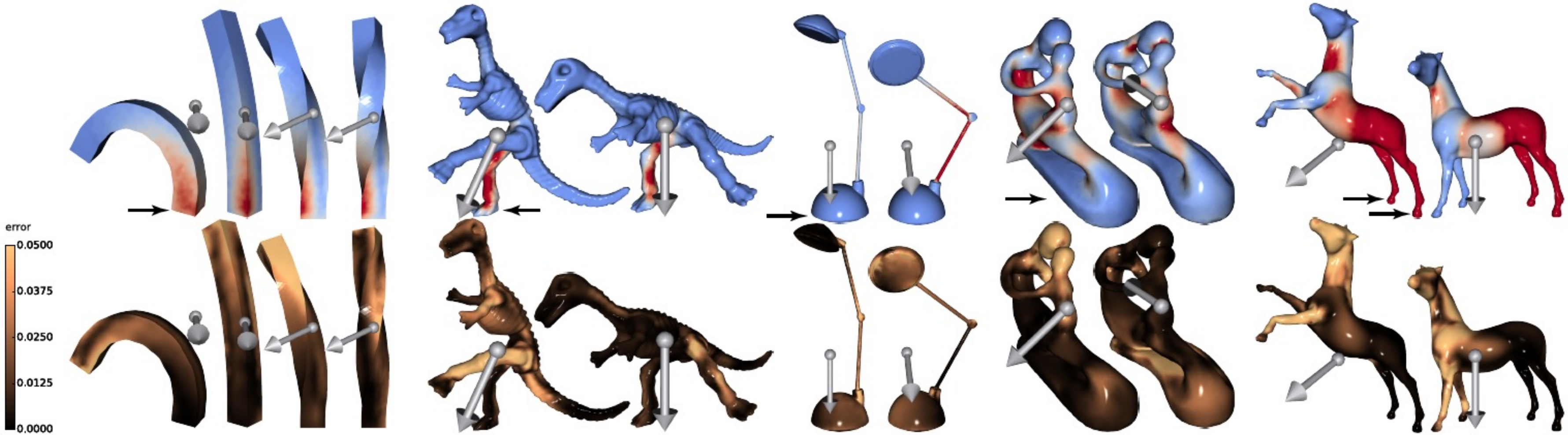


estimation

ground truth

Results

Accuracy – visualization



Results

Efficiency

Model	Estimation Timings (sec)		
	Full	Est (SU)	SpEst (SU)
bar	1.9	0.36 (5×)	0.026 (73×)
horse	12.4	0.89 (14×)	0.071 (175×)
bird	16.6	1.17 (14×)	0.083 (200×)
dinosaur	3.8	0.65 (6×)	0.042 (90×)
fertility	22.6	1.73 (13×)	0.165 (137×)
lamp	3.9	2.34 (2×)	0.158 (25×)
gargoyle	6.7	2.23 (3×)	0.153 (44×)
armadillo	27.9	11.04 (3×)	0.419 (67×)

Conclusion and Future Work

Example-based subspace stress analysis method

Accurate stress estimation

Efficient runtime

Predefined boundary conditions

Valid design scope

Acknowledgments

Valuable comments from anonymous reviewers

Open source libraries

libigl, Vega FEM

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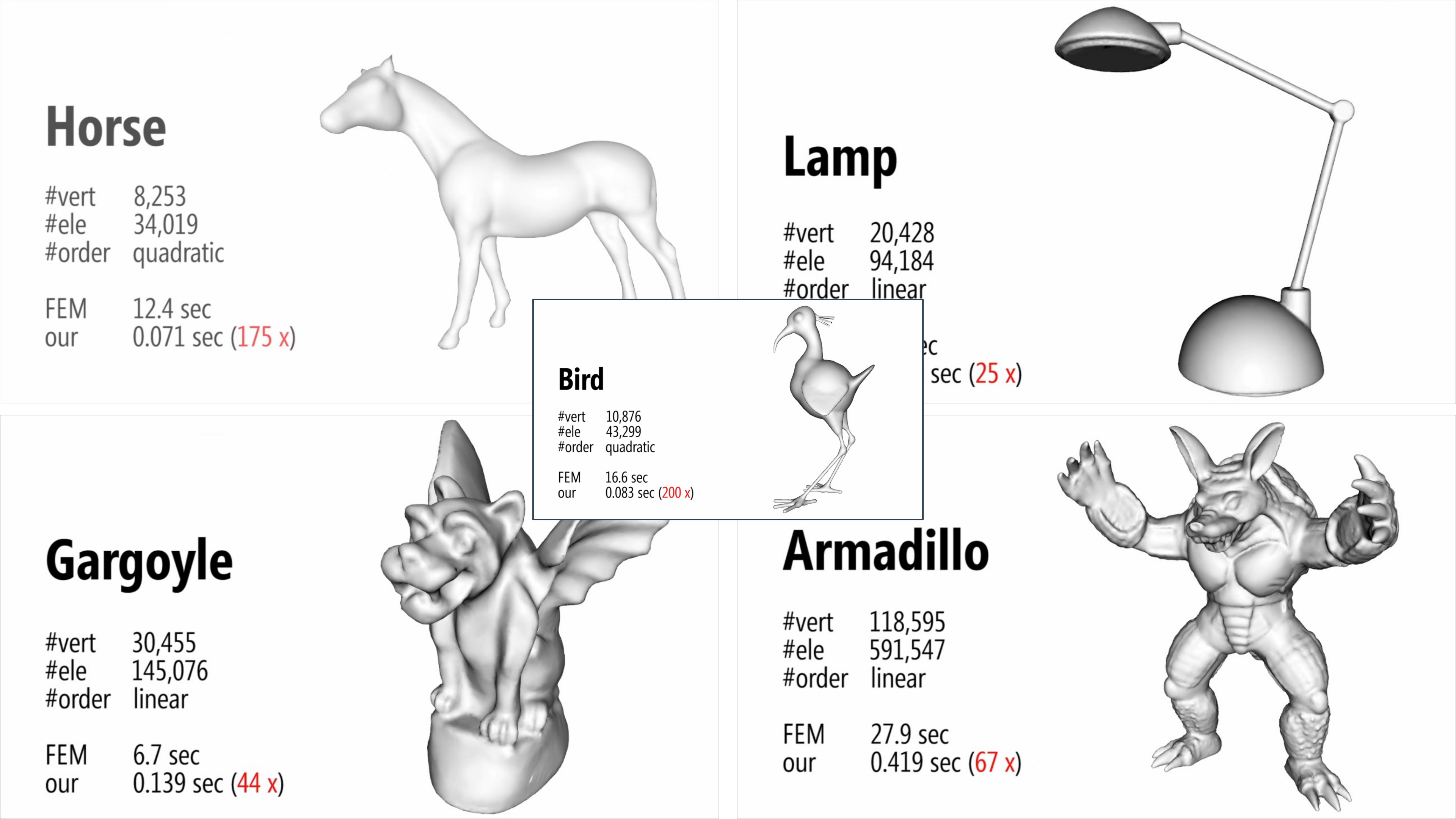

Horse
#vert 8,253
#ele 34,019
#order quadratic
FEM 12.4 sec
our 0.071 sec (175 x)
Lamp
#vert 20,428
#ele 94,184
#order linear
ec
sec (25 x)
Bird
#vert 10,876
#ele 43,299
#order quadratic
FEM 16.6 sec
our 0.083 sec (200 x)
Gargoyle
#vert 30,455
#ele 145,076
#order linear
FEM 6.7 sec
our 0.139 sec (44 x)
Armadillo
#vert 118,595
#ele 591,547
#order linear
FEM 27.9 sec
our 0.419 sec (67 x)