

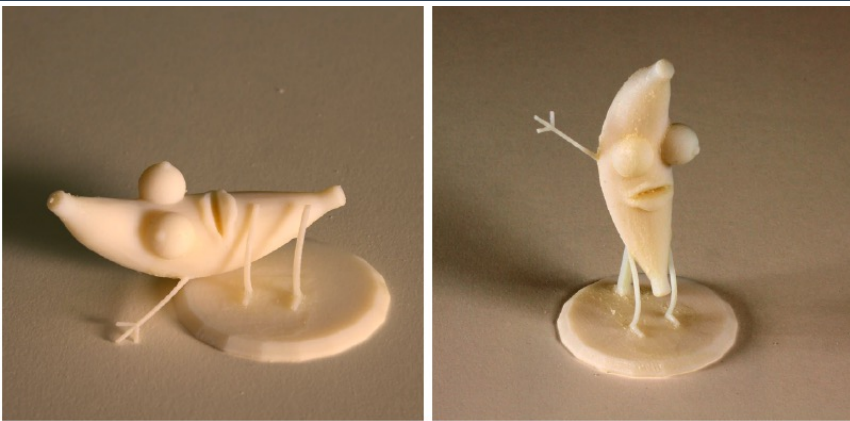
# Example-Based Subspace Stress Analysis for Interactive Shape Design

IEEE TVCG 2016

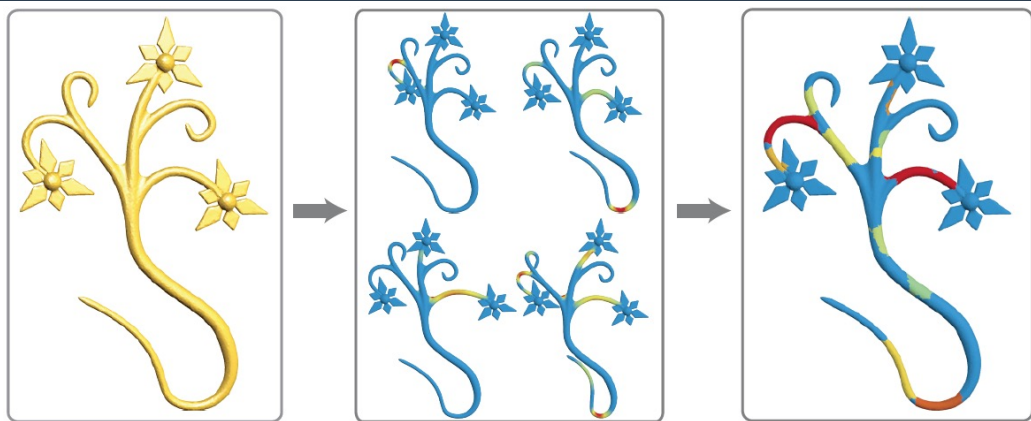
Xiang Chen   Changxi Zheng   Kun Zhou  
Zhejiang University and Columbia University



# Stress Analysis



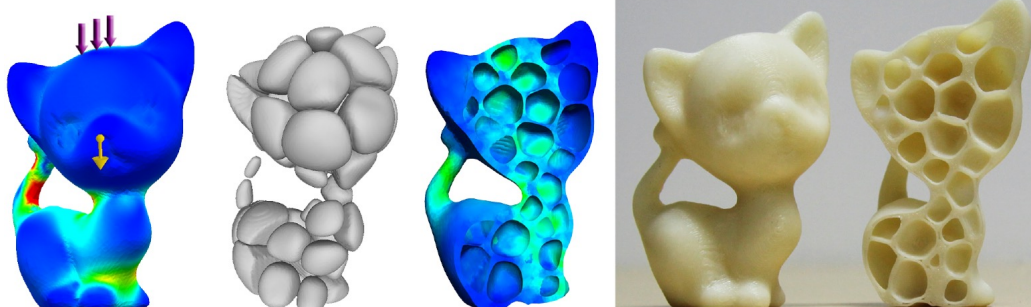
[Stava et al. 2012]



[Zhou et al. 2013]

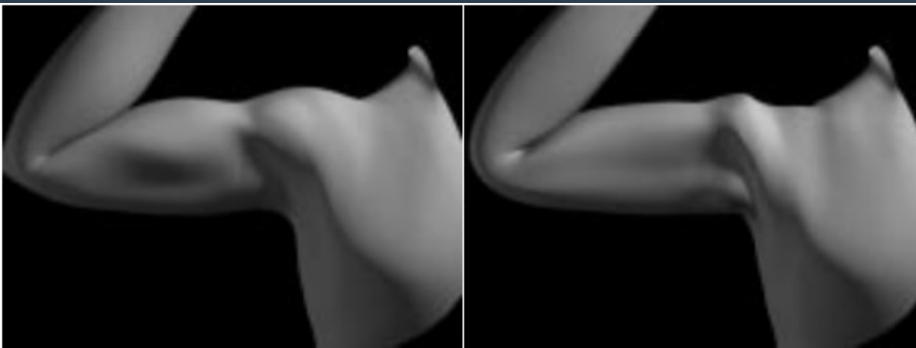


[Wang et al. 2013]

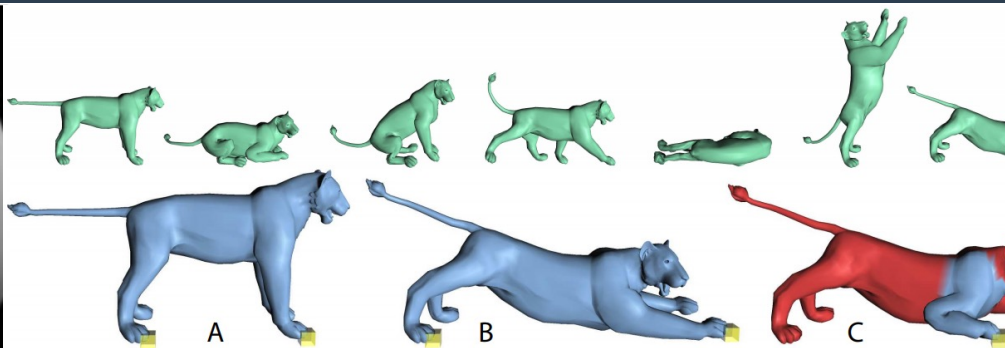


[Lu et al. 2014]

# Subspace Shape Deformation



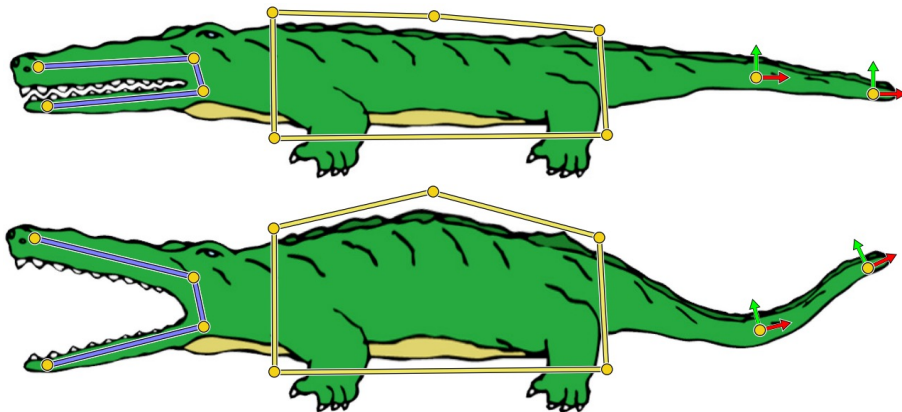
[Lewis et al. 2000]



[Sumner et al. 2005]



[Weber et al. 2007]

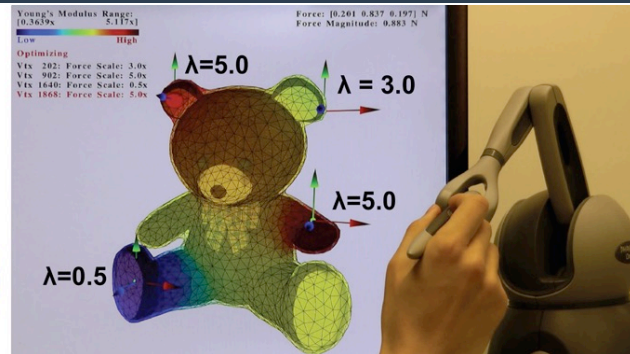


[Jacobson et al. 2011]

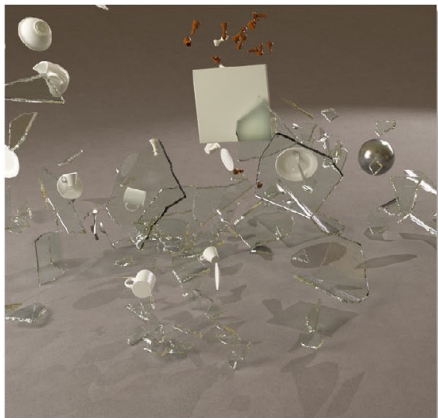
# Subspace Dynamic Simulation



[Martin et al. 2011]



[Xu et al. 2015]



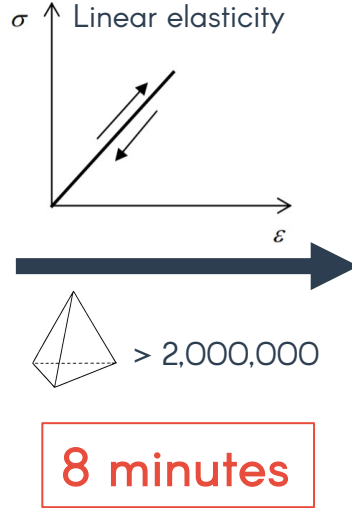
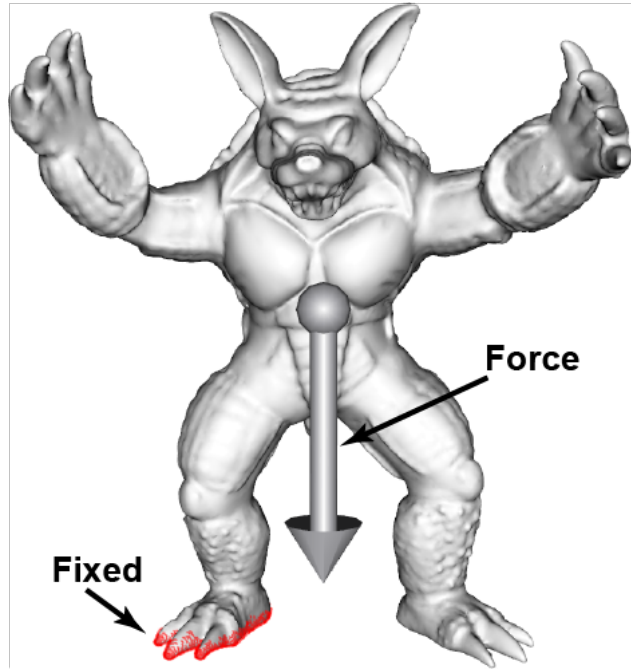
[Zheng et al. 2010]



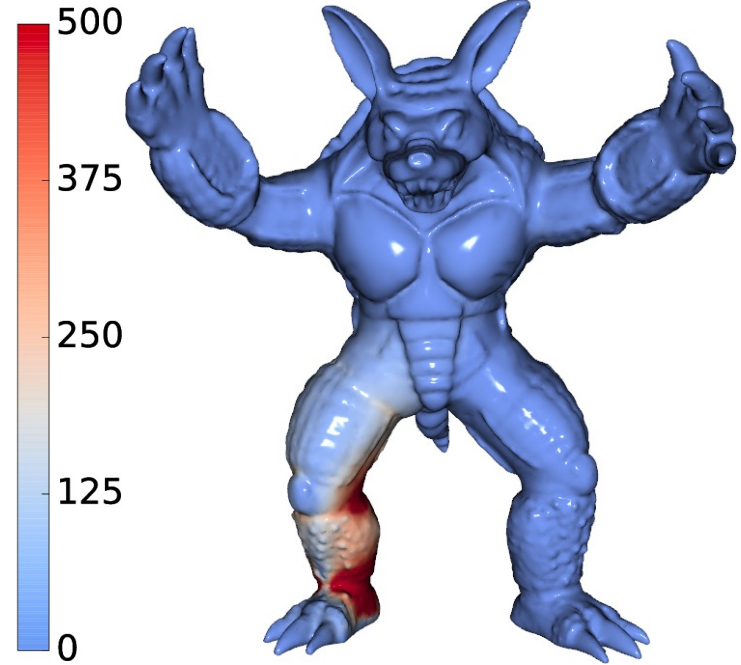
[Chai et al. 2014]



# Background Concepts

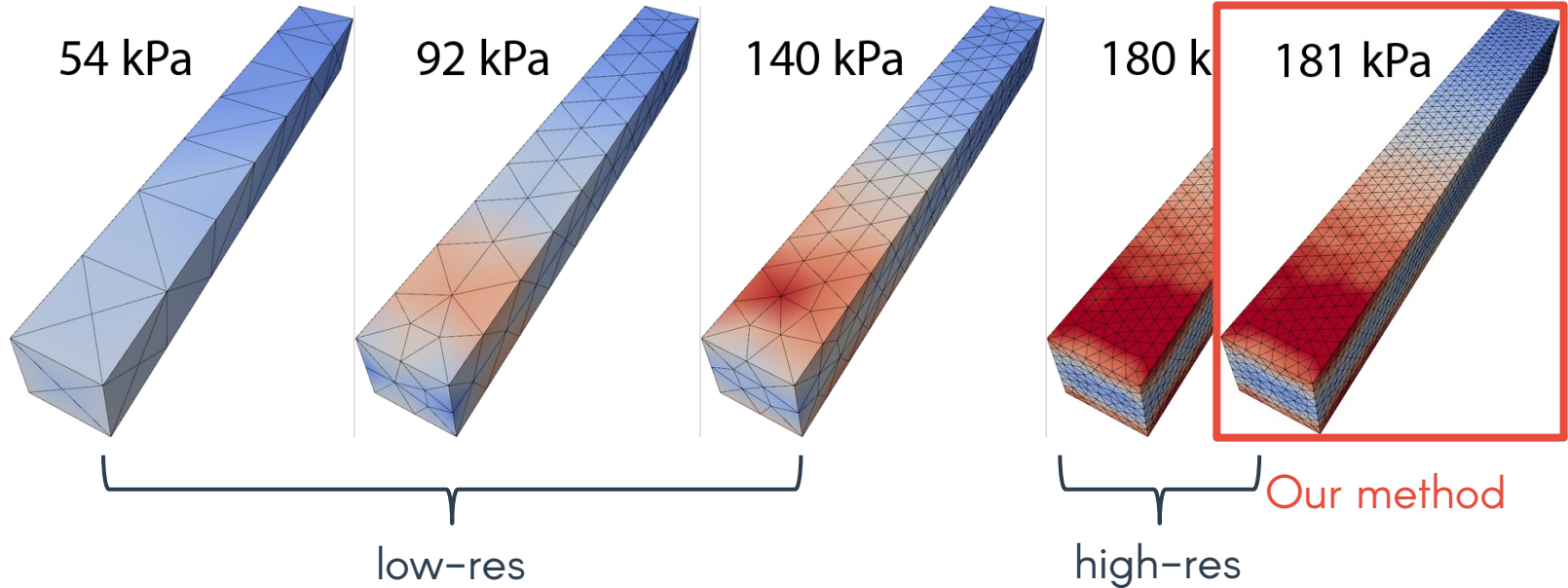


von Mises (kPa)  
max:4733



# Reduced Stress Analysis

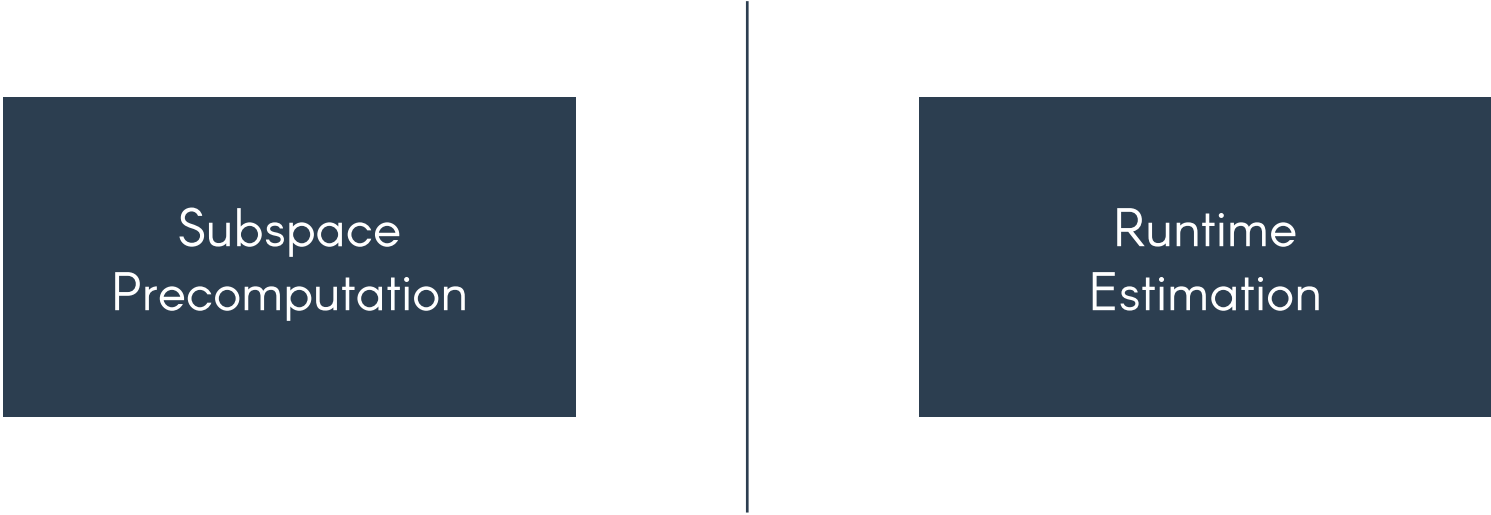
Low resolution mesh



# Reduced Stress Analysis

Rest shapes correlated

Two phase strategy

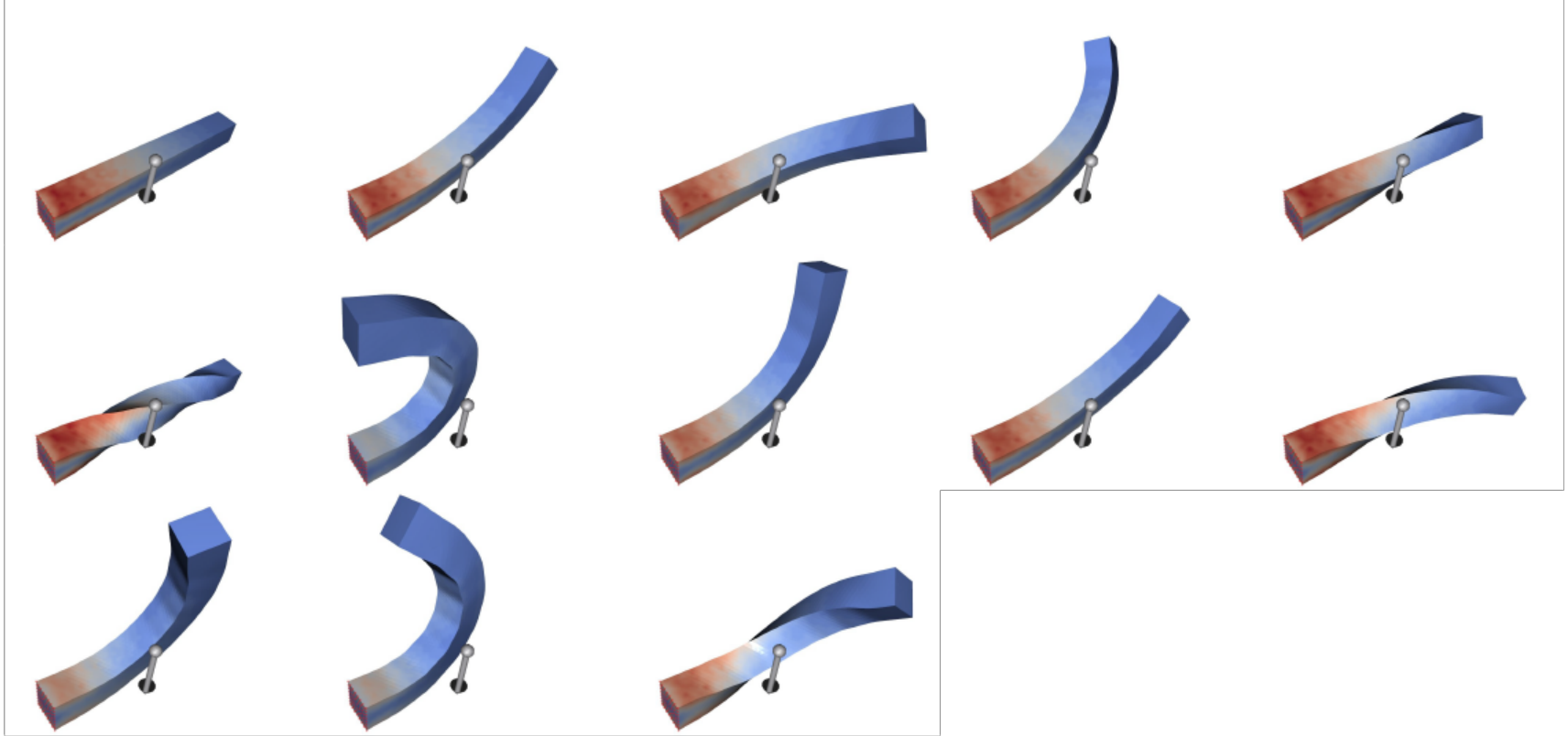


The diagram illustrates a two-phase strategy for reduced stress analysis. It consists of two dark blue rectangular boxes, one on the left and one on the right, separated by a thin vertical line. The left box contains the text 'Subspace Precomputation' and the right box contains the text 'Runtime Estimation'.

Subspace  
Precomputation

Runtime  
Estimation

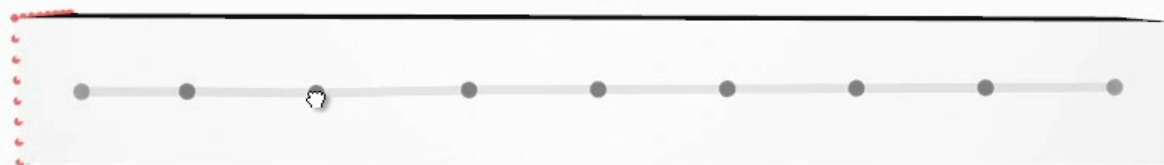
# Reduced Stress Analysis







73 x speedup over FEM



# Reduced Stress Analysis

Subspace  
Precomputation

Runtime  
Estimation

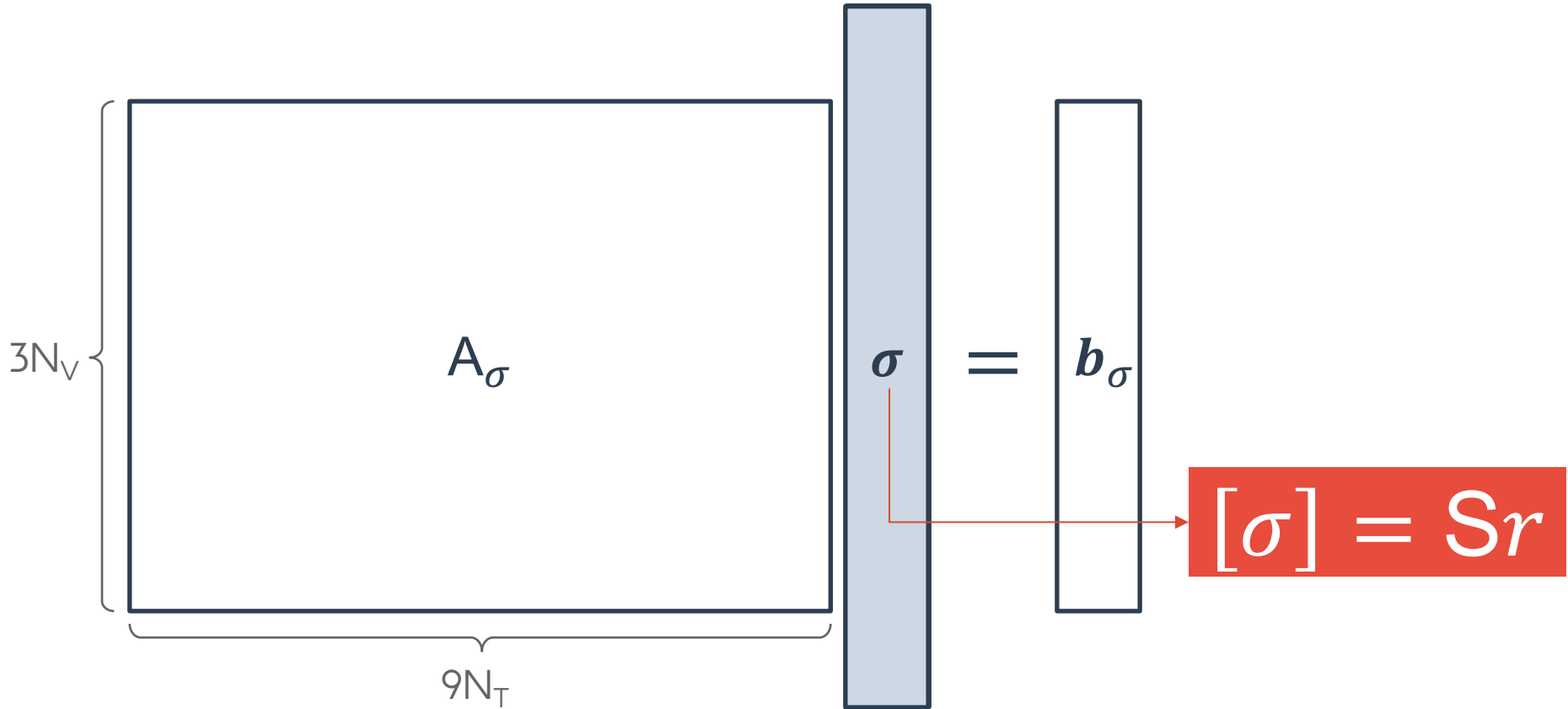
# Reduced Stress Analysis

Standard FEM discretizes the displacement,  $\boldsymbol{u}$

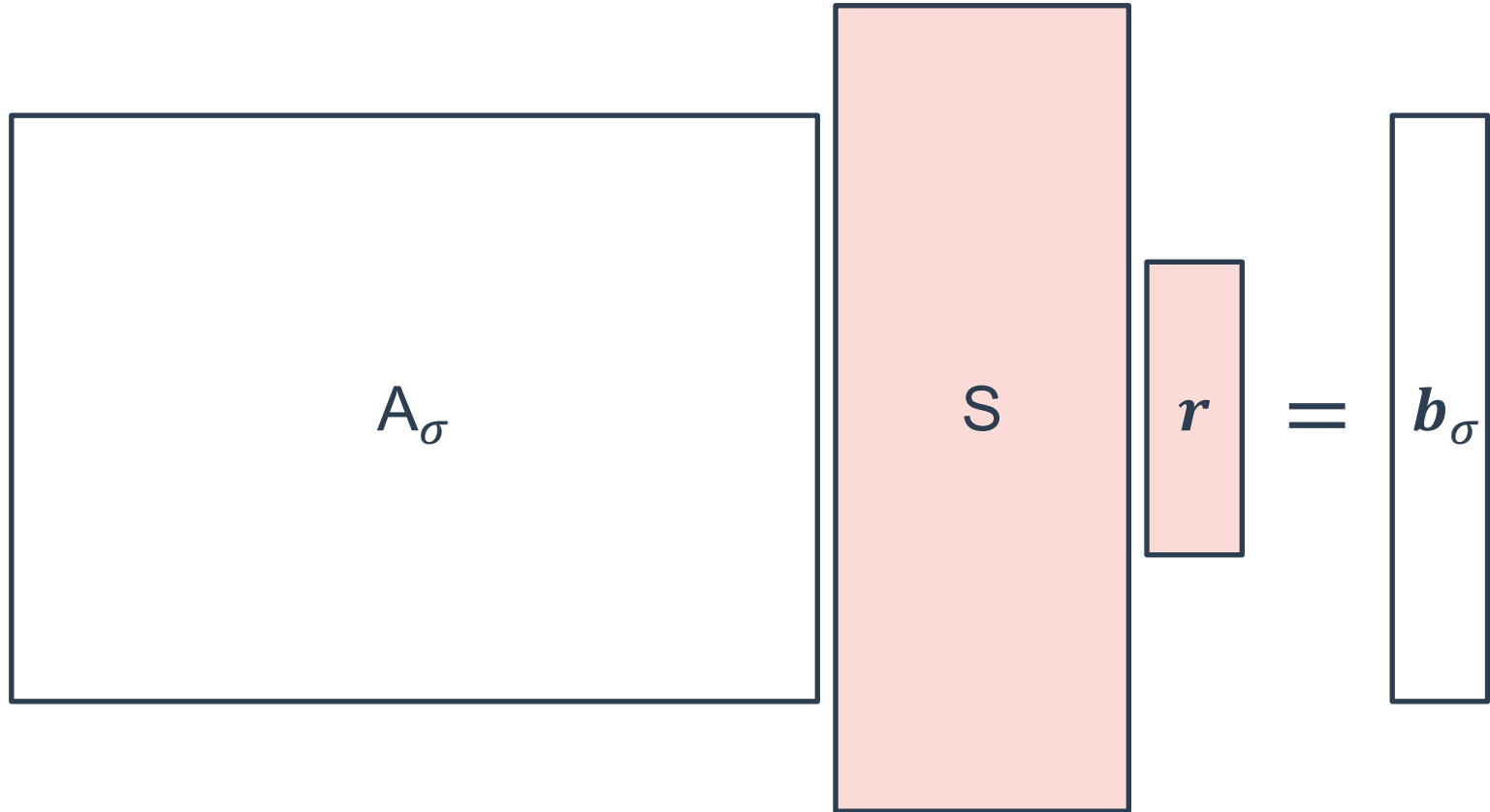
Instead, discretizing the weak form w.r.t. stress,  $\boldsymbol{\sigma}$

$$\boldsymbol{A}_\sigma[\boldsymbol{\sigma}] = \boldsymbol{b}_\sigma$$

# Reduced Stress Analysis



# Reduced Stress Analysis



# Reduced Stress Analysis

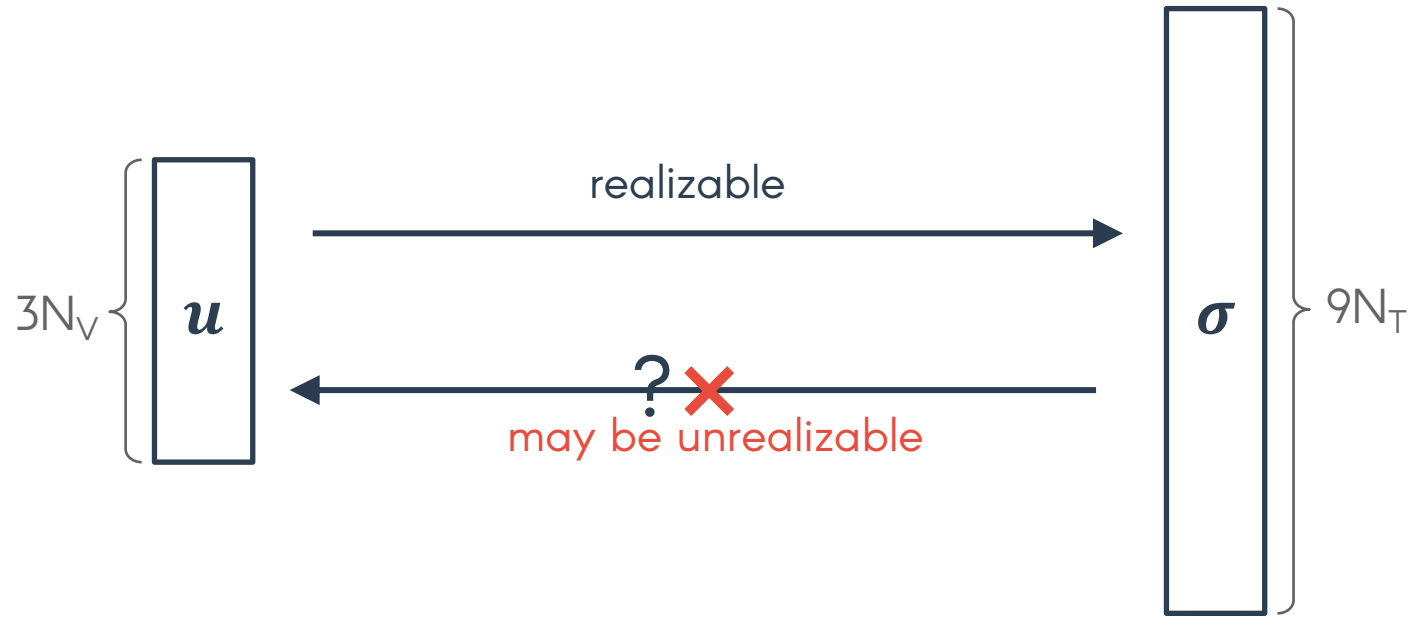
The diagram illustrates the reduced stress analysis equation:  $A_\sigma S r = b_\sigma$ . The matrix  $A_\sigma S$  is represented by a large white rectangle with a vertical brace on its left indicating a dimension of  $3N_v$  and a horizontal brace at its bottom indicating a dimension of  $N_r$ . The vector  $r$  is a smaller, shaded blue rectangle. The vector  $b_\sigma$  is a tall, white rectangle. The equation is shown as  $A_\sigma S r = b_\sigma$ .

Small-scale least-squares problem



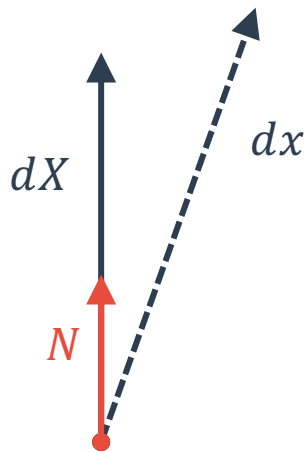
# Reduced Stress Analysis

Physically realizable stress?



# Reduced Stress Analysis

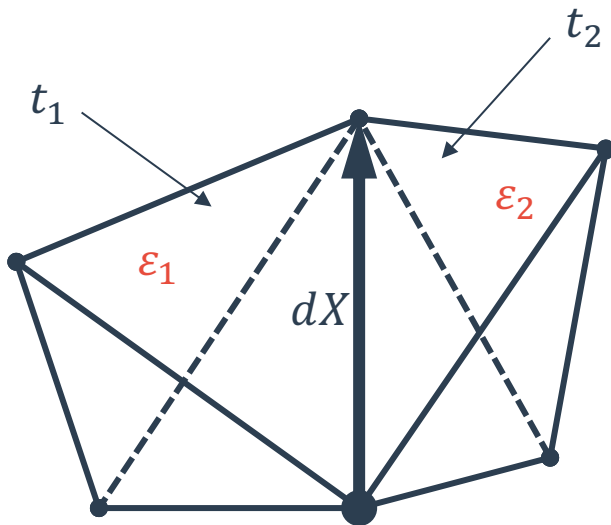
Regularization



$$N^T \epsilon N = \frac{\|dx\|_2 - \|dX\|_2}{\|dX\|_2}$$

# Reduced Stress Analysis

Regularization



$$N^T \epsilon_1 N = N^T \epsilon_2 N$$

# Reduced Stress Analysis

Quadratic minimization

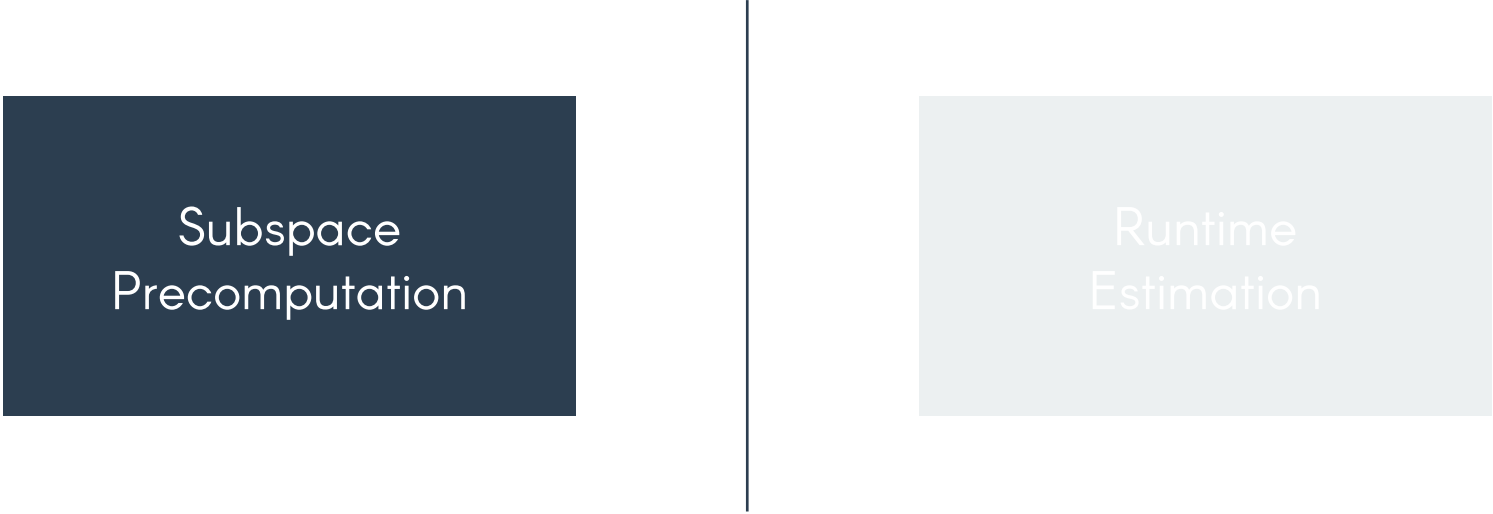
$$\min_{\mathbf{r}} \|(\mathbf{A}_\sigma \mathbf{S})\mathbf{r} - \mathbf{b}_\sigma\|_2^2 + \alpha E_r(\mathbf{r})$$

# Reduced Stress Analysis

Quadratic minimization

$$\min_{\mathbf{r}} \underbrace{\|(\mathbf{A}_\sigma \mathbf{S})\mathbf{r} - \mathbf{b}_\sigma\|_2^2}_{\text{Force term}} + \overset{\text{Weight factor}}{\uparrow} \underbrace{\alpha E_r(\mathbf{r})}_{\text{Regularization term}}$$

# Reduced Stress Analysis



The diagram illustrates the 'Reduced Stress Analysis' process. It features a dark blue header at the top. Below the header, a vertical line divides the space into two columns. The left column contains a dark blue rectangular box with the text 'Subspace Precomputation'. The right column contains a light gray rectangular box with the text 'Runtime Estimation'.

Subspace  
Precomputation

Runtime  
Estimation

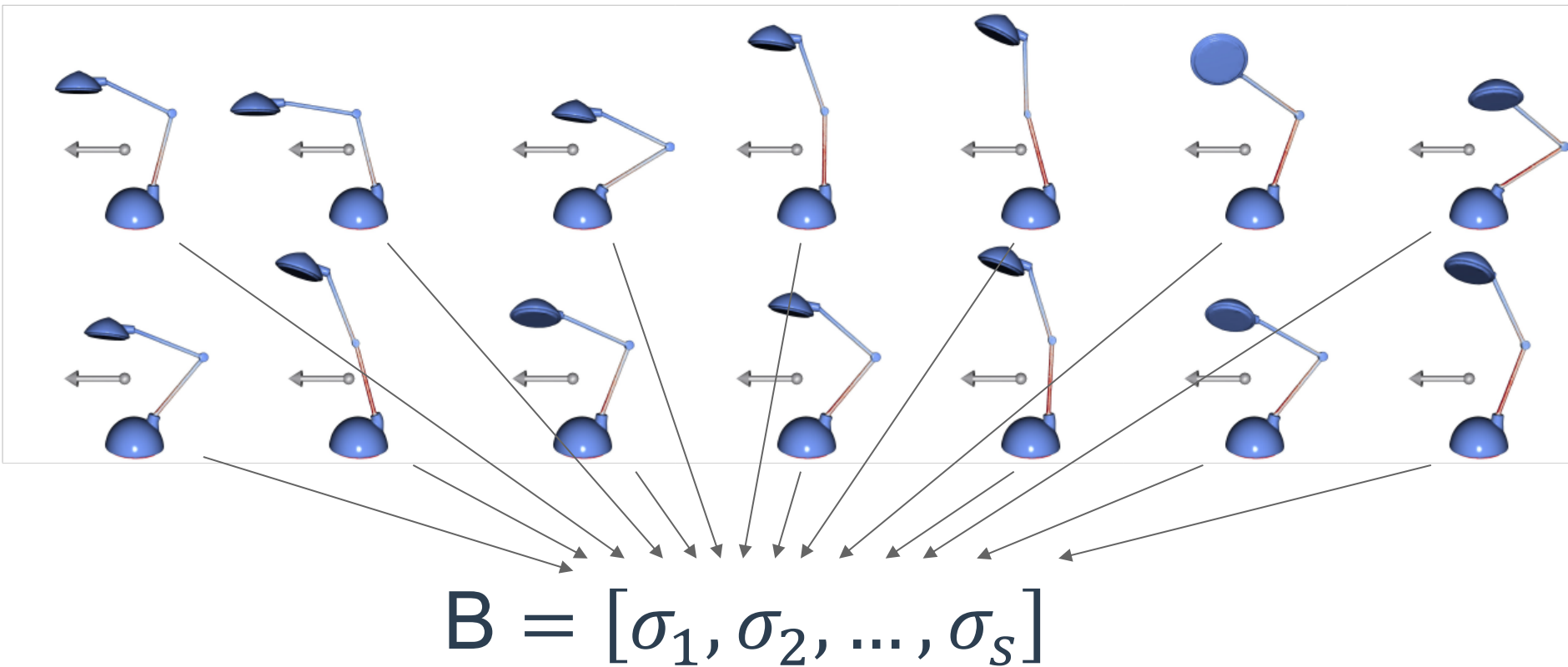


# Construction of Stress Basis

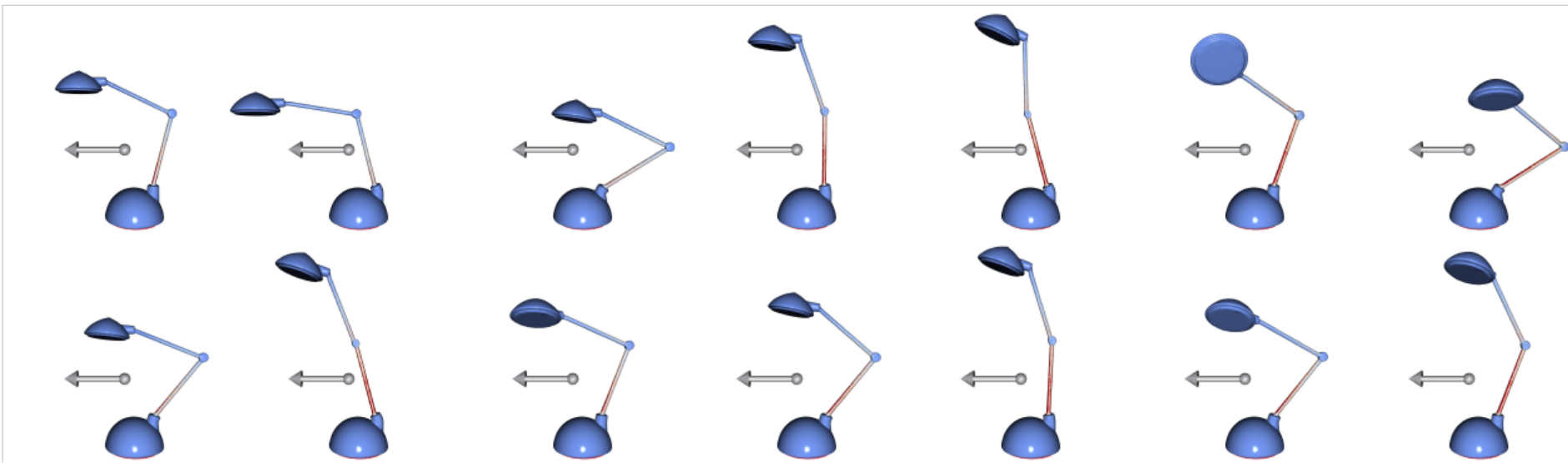
How to compute?

$$[\sigma] = \underset{\uparrow}{S} r$$

# Construction of Stress Basis

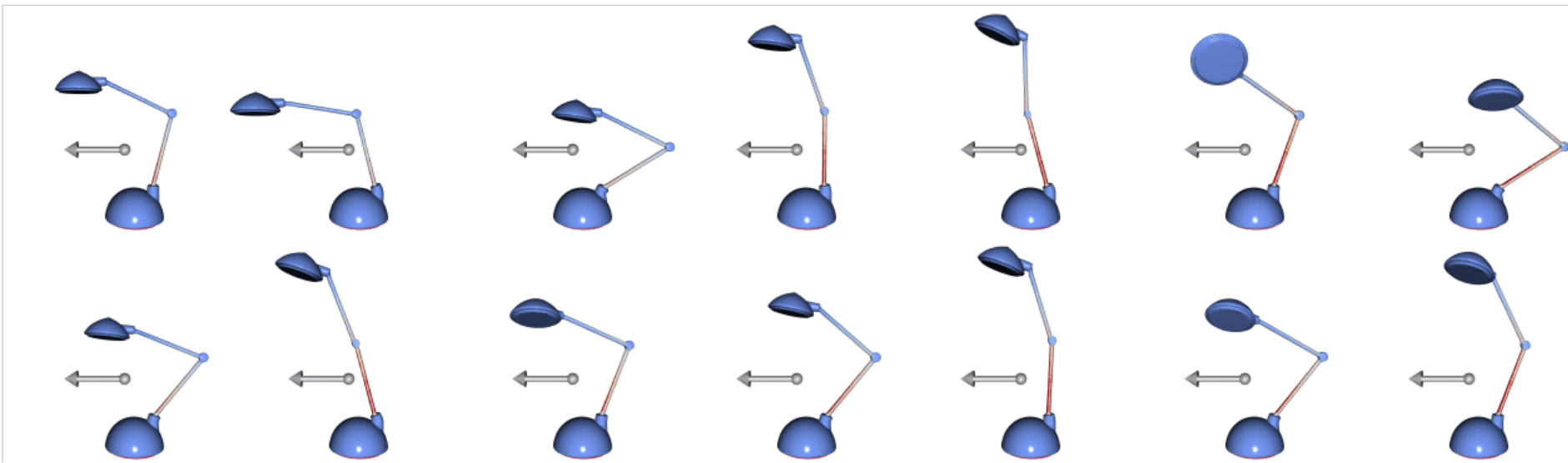


# Construction of Stress Basis



$$S = \text{PCA}(B)$$

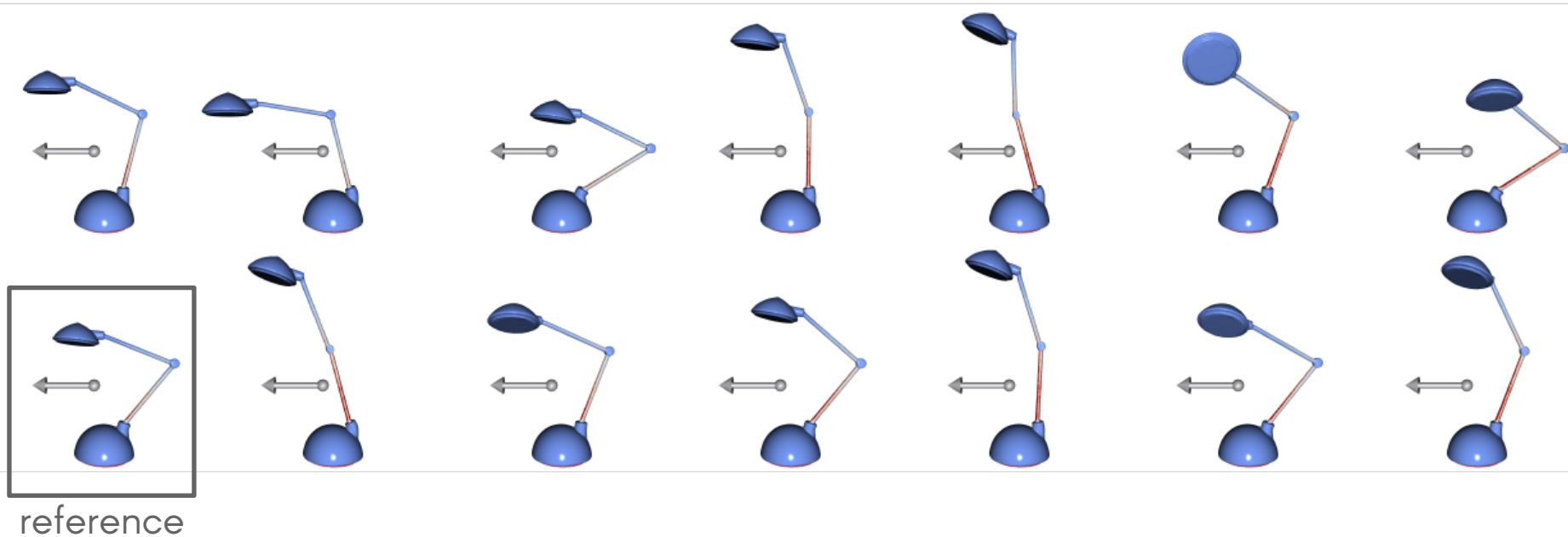
# Construction of Stress Basis



$$S = \text{PCA}(\mathbf{B})$$

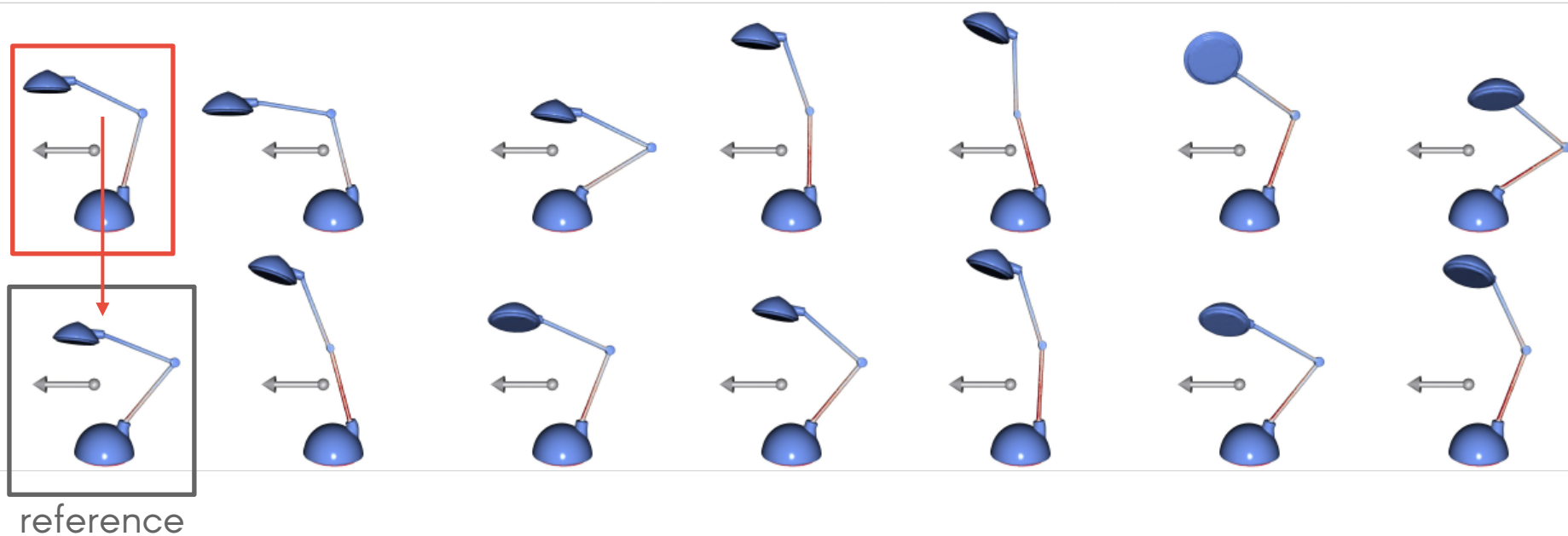
$$\rightarrow \mathbf{pb}(\mathbf{B})$$

# Construction of Stress Basis



$$S = \text{PCA}([\sigma_1, \sigma_2, \dots, \sigma_s])$$

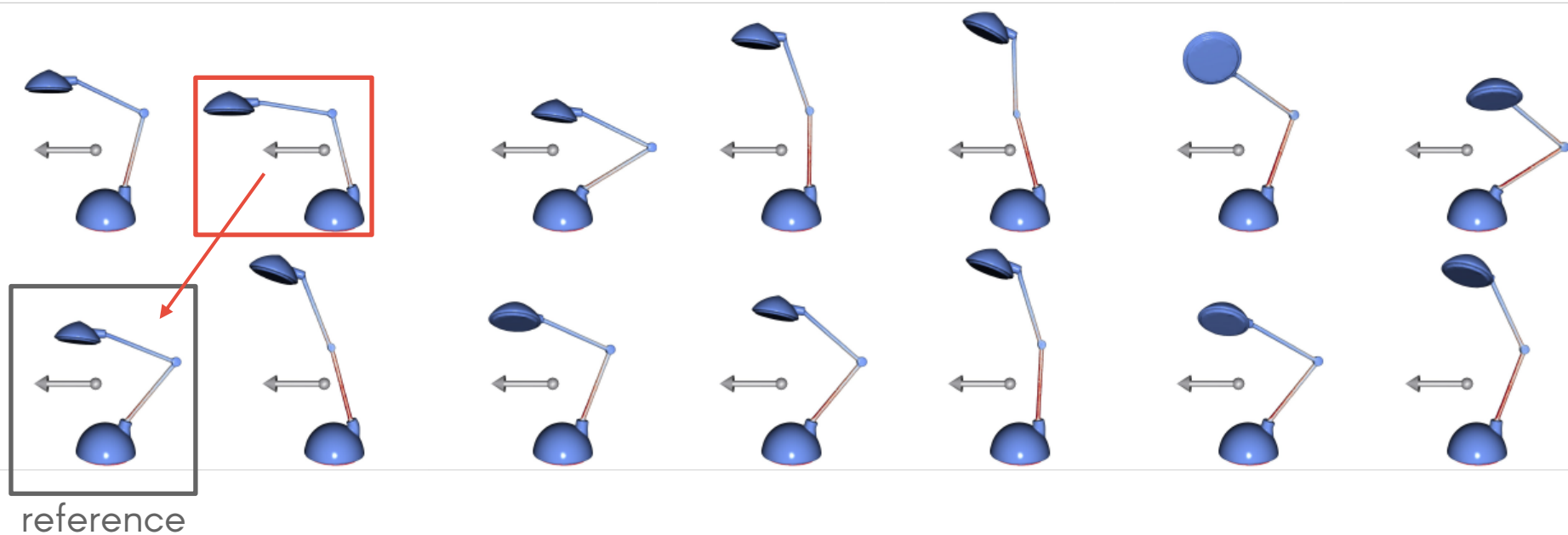
# Construction of Stress Basis



$$S = \text{PCA}([pb(\sigma_1), \sigma_2, \dots, \sigma_s])$$

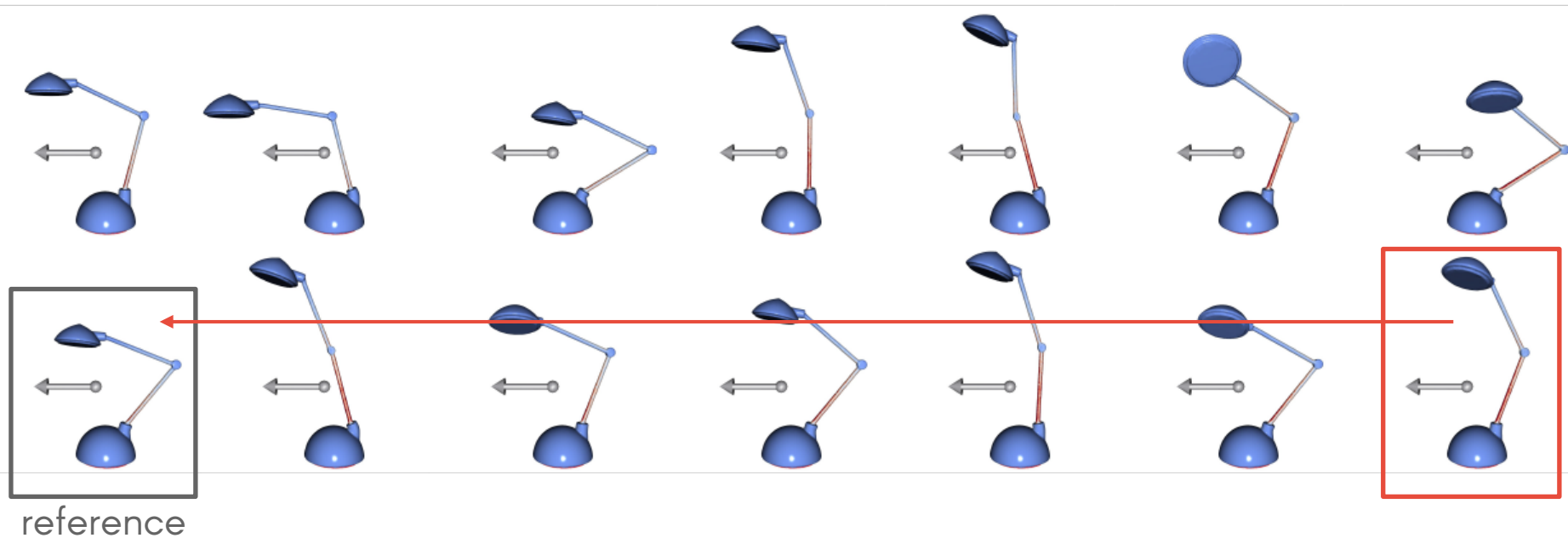


# Construction of Stress Basis



$$S = \text{PCA}([pb(\sigma_1), pb(\sigma_2), \dots, \sigma_s])$$

# Construction of Stress Basis



$$S = \text{PCA}([pb(\sigma_1), pb(\sigma_2), \dots, pb(\sigma_s)])$$

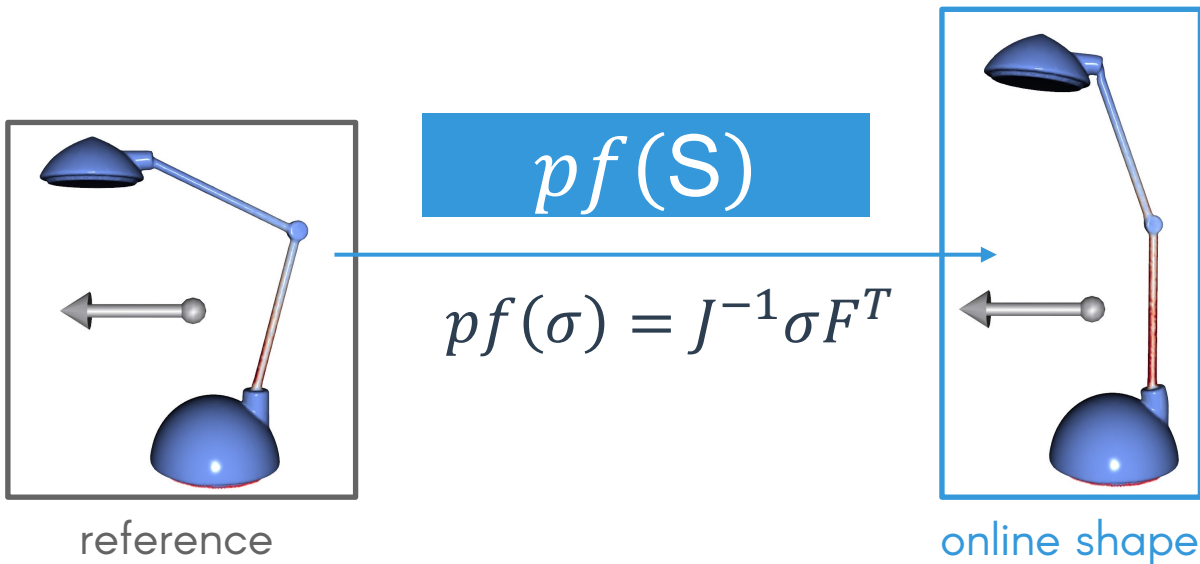
# Construction of Stress Basis

Stress tensor pull-back (inspired from the first PK tensor)

$$pb(\sigma) = J\sigma F^{-T}$$

# Construction of Stress Basis

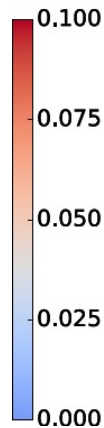
Stress tensor push-forward



# Construction of Stress Basis

Efficacy of pull-back and push-forward

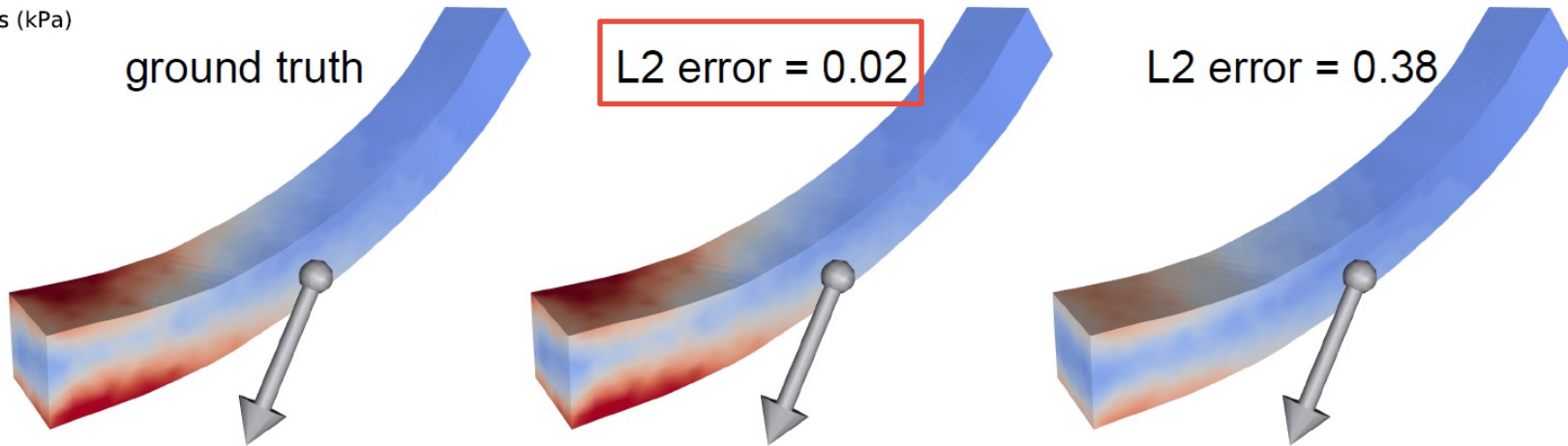
von Mises (kPa)



ground truth

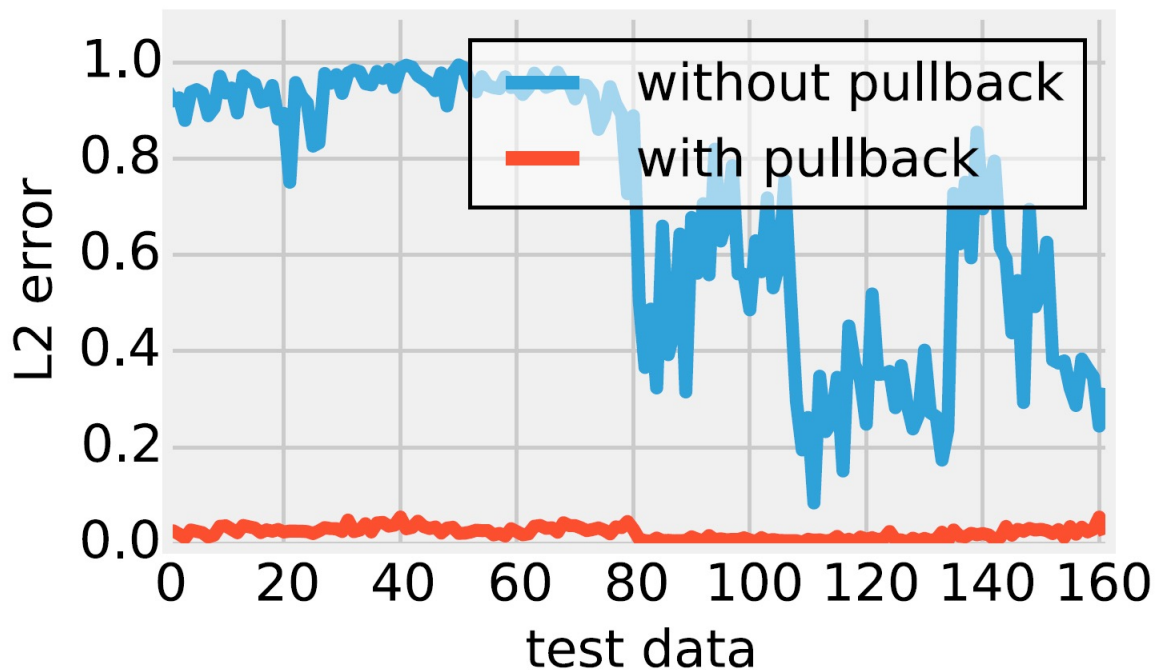
L2 error = 0.02

L2 error = 0.38



# Construction of Stress Basis

Efficacy of pull-back and push-forward





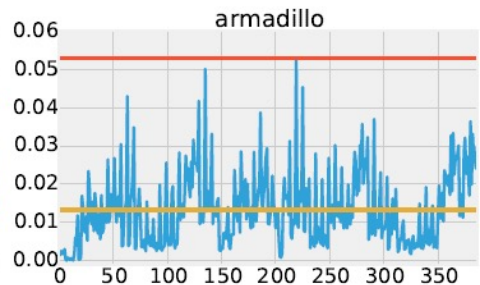
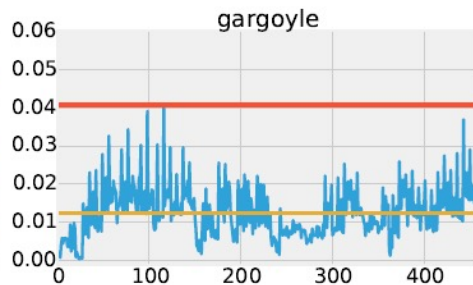
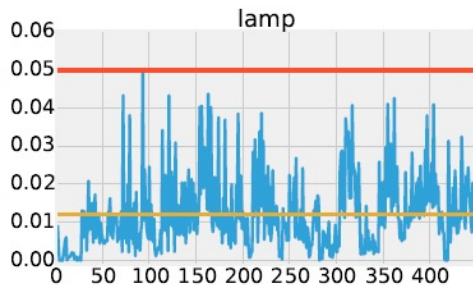
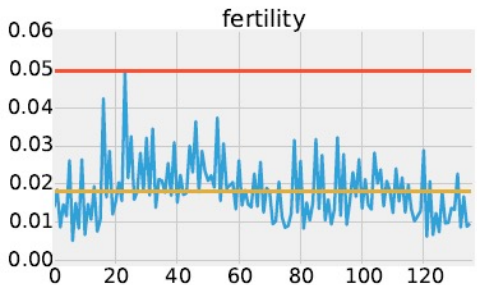
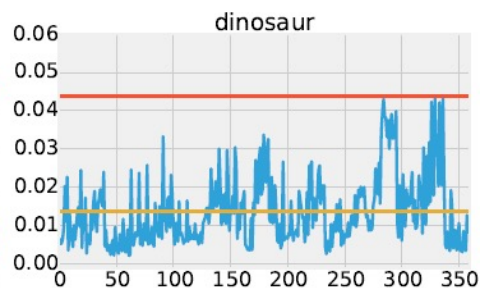
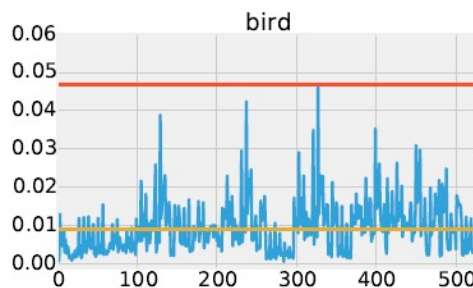
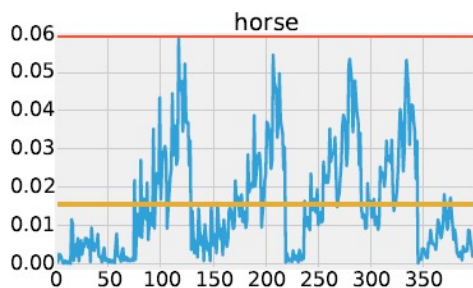
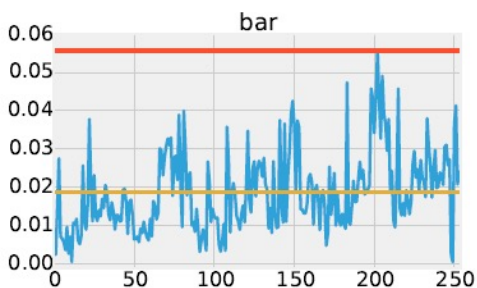
# Results

Training and test data: 8 groups

Model	#Vertices	#Elements	#Order	#DoFs	#Train	#Test
bar	3,452	15,602	linear	10,356	39	254
horse	8,253	34,019	quadratic	167,034	21	399
bird	10,876	43,299	quadratic	216,072	27	525
dinosaur	14,029	57,830	linear	42,087	33	359
fertility	17,492	82,271	quadratic	377,667	18	136
lamp	20,428	94,184	linear	61,284	42	448
gargoyle	30,455	145,076	linear	91,365	33	457
armadillo	118,595	591,547	linear	355,785	33	387

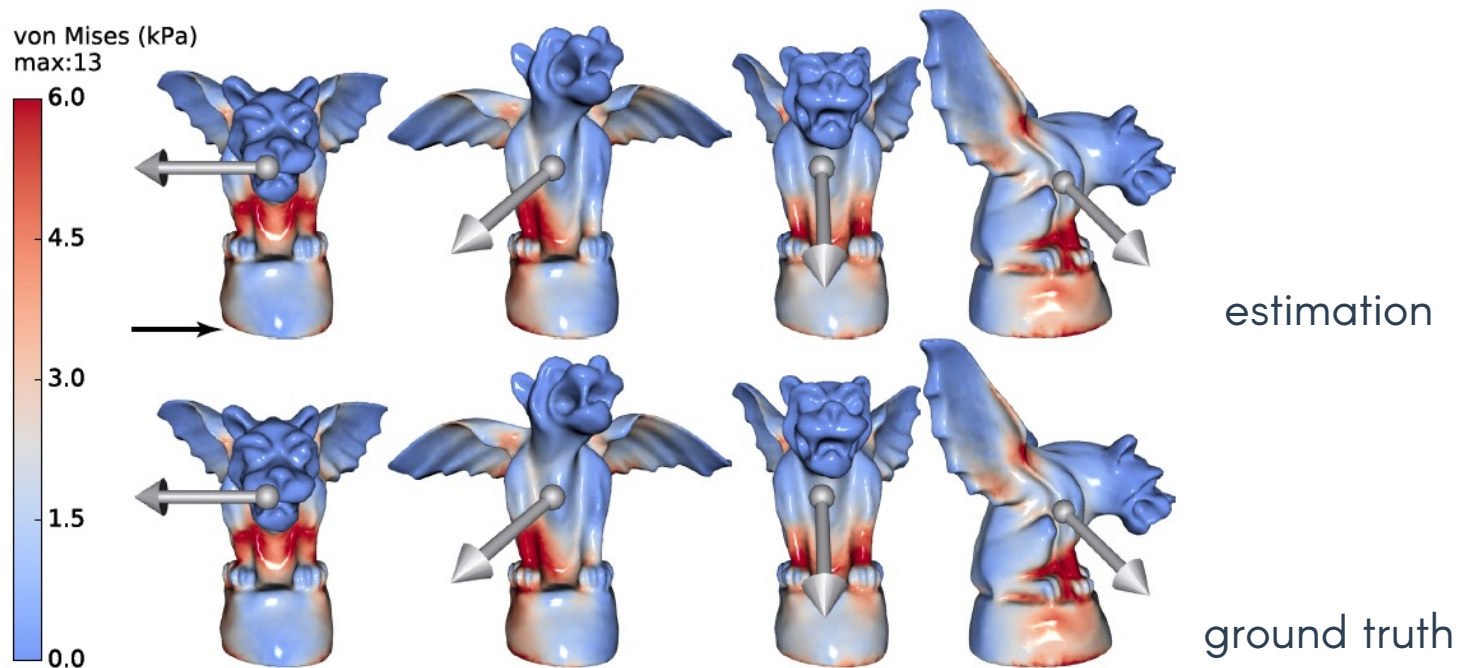
# Results

Accuracy –  $L_2$  errors: max < 6%, average < 2%



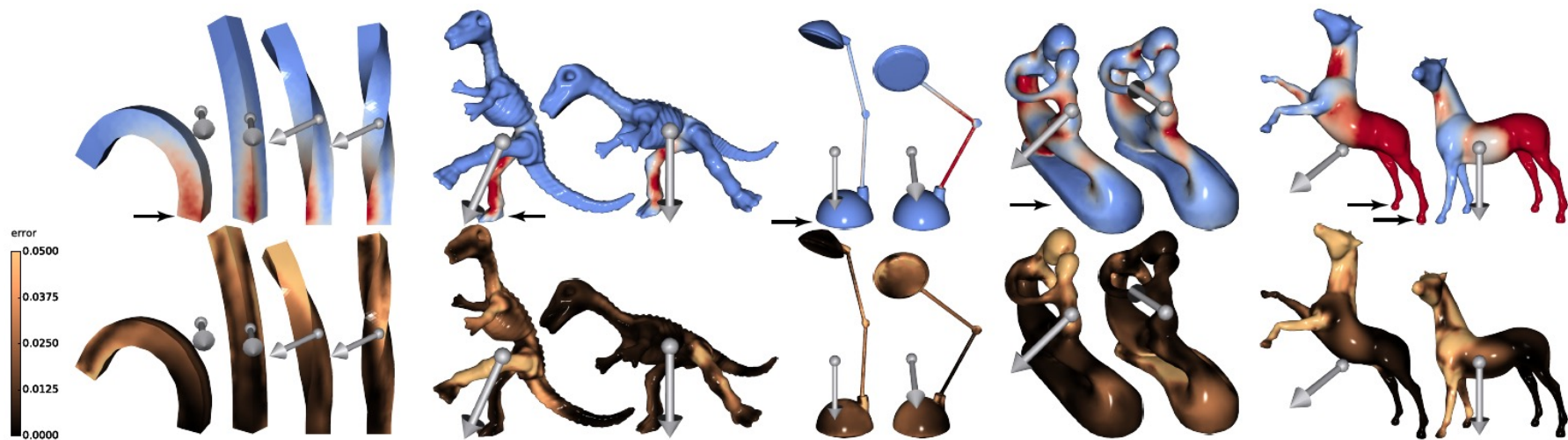
# Results

## Accuracy – visualization



# Results

## Accuracy – visualization



# Results

## Efficiency

Model	Estimation Timings (sec)		
	Full	Est (SU)	SpEst (SU)
bar	1.9	0.36 (5×)	0.026 (73×)
horse	12.4	0.89 (14×)	0.071 (175×)
bird	16.6	1.17 (14×)	0.083 (200×)
dinosaur	3.8	0.65 (6×)	0.042 (90×)
fertility	22.6	1.73 (13×)	0.165 (137×)
lamp	3.9	2.34 (2×)	0.158 (25×)
gargoyle	6.7	2.23 (3×)	0.153 (44×)
armadillo	27.9	11.04 (3×)	0.419 (67×)

# Conclusion and Future Work

Example-based subspace stress analysis method

Accurate stress estimation

Efficient runtime

Predefined boundary conditions

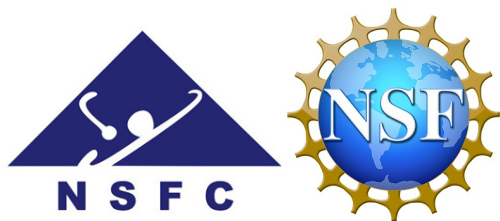
Valid design scope

# Acknowledgments

Valuable comments from anonymous reviewers

Open source libraries  
libigl, Vega FEM

Funding: NSF of China (No.61303136 and No.61272305), National Science Foundation of U.S. (CAREER-1453101)

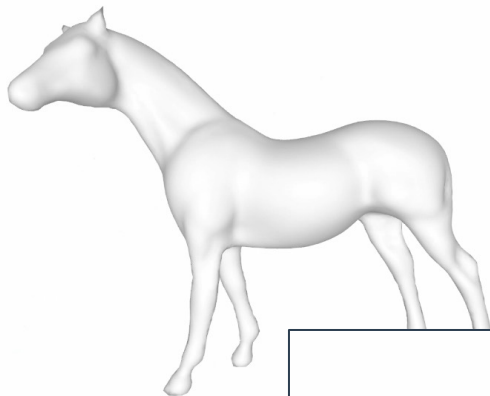




# Horse

#vert 8,253  
#ele 34,019  
#order quadratic

FEM 12.4 sec  
our 0.071 sec (175 x)



# Lamp

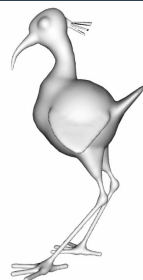
#vert 20,428  
#ele 94,184  
#order linear

ec  
sec (25 x)



# Bird

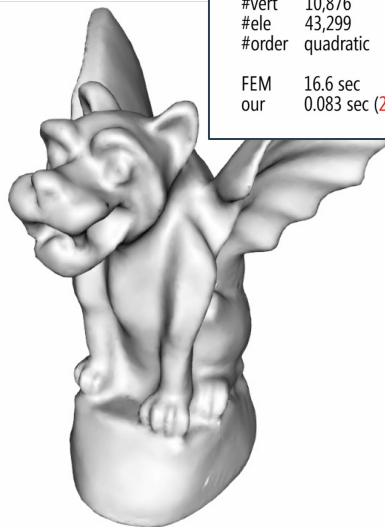
#vert 10,876  
#ele 43,299  
#order quadratic  
FEM 16.6 sec  
our 0.083 sec (200 x)



# Gargoyle

#vert 30,455  
#ele 145,076  
#order linear

FEM 6.7 sec  
our 0.139 sec (44 x)



# Armadillo

#vert 118,595  
#ele 591,547  
#order linear

FEM 27.9 sec  
our 0.419 sec (67 x)

